ECH3301 Process Analysis and Design Fall 2010

HW 6 Laplace Transform Due Nov 3

1. Find

$$\mathscr{L}^{-1}\left\{\frac{2s-4}{(s^2+s)(s^2+1)}\right\},$$

$$\mathscr{L}\{t\int_0^t \sin\tau d\tau\},$$

$$\mathscr{L}^{-1}\left\{\frac{s}{s^2-10s+29}\right\},$$

$$\mathscr{L}^{-1}\left\{\frac{s+\pi}{s^2+\pi^2}e^{-s}\right\}.$$

2. **(Mass-spring system)** Suppose a mass weighing 32 lb stretches a spring 2 ft. If the weight is released from rest at the equilibrium position, find the equation of motion $x(t)$ if an impressed force $f(t) = 20t$ acts on the system for $0 \le t < 5$ and is then removed. Ignore any damping forces.

3. **(Beam)** A uniform beam of length L carries a concentrated load w_0 at $x = \frac{1}{2}$. The beam is embedded at its left and is free at its right end. Use the Laplace transform to determine the deflection $y(x)$ from

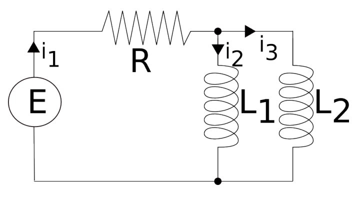
$$EI\frac{d^4y}{dx^4} = w_0\,\delta(x - \frac{1}{2}L),$$

where $y(0) = 0, y'(0) = 0, y''(L) = 0$, and $y'''(L) = 0$.

[Hint: The constant EI is called flexural rigidity. The manner in which the beam is supported at the ends determines the form of the boundary conditions. For more information on beam applications, consult Schaum's book or any other relevant text.]

4. **(RLC circuit)** The system of differential equations for the currents $i_2(t)$ and $i_3(t)$ in the electrical network shown below is

$$L_1\frac{di_2}{dt} + Ri_2 + Ri_3 = E(t)$$
$$L_2\frac{di_3}{dt} + Ri_2 + Ri_3 = E(t).$$



Solve the system if R = 5 Ω, $L_1 = 0.01$ h, $L_2 = 0.0125$ h, $E = 100$ V, $i_2(0) = 0$ and $i_3(0) = 0$. Also determine the current $i_1(t)$.

[Hint: Use Kirchhoff's law to determine i_1. For more information on circuit applications, consult Schaum's book or any other relevant text.]