ECH3301 Process Analysis and Design 12/08/10

Final Exam Name _____

This is a closed book exam. Calculators are allowed. You can use one 8.5×11 sheet of paper (both sides) with notes of your choice.

1. A missile is fired and its position in flight in observed by a radar at the following positions:

Position down range (1000 miles)	0	0.25	0.5	0.75	1
Height (1000 miles)	0	0.008	0.015	0.019	0.02

   Supposing the missile is programmed to follow a parabolic flight path, predict how far down range the missile will land.

   [Hint: Determine the parabola $f(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2$ that best fits the observed data in the least squares sense.]

2. The temperature in a room is 70 °F. A thermometer which has been kept in it is placed outside. In 5 min the thermometer reading is 60 °F. Five minutes later, it is 55 °F. Find the outdoor temperature.

   [Hint: Use the Newton's law of cooling.]

3. A capacitor whose capacitance is $C = 2 \times 10^{-3}$ farad, an inductor with inductance $L = 1/20$ henry, and a resistor with resistance $R = 1$ ohm are connected in series, along with the battery of an electromotive force $E = 50\sin(120t)$. If at $t = 0$ seconds, there is no charge q on the capacitor and no current i in the circuit, find the steady state current.

   [Hint: Since the current (in amperes) is $i = \frac{dq}{dt}$, where q is the charge (in coulombs), the steady state current $i_{ss} = i_p$ follows from the particular solution q_p of $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t)$.]

4. A particle of mass 2 grams moves on the x axis and is attracted toward the origin with a force equal to $8x$. If it is initially at rest at $x = 10$, find the position of the particle at any time if an external force $F(t) = F_0 U(t - a)$ acts on the particle, but there is no damping force. F_0 and a are constants and $U(t)$ is a step function.

5. Using the matrix-eigenvalue approach, solve the following system of ODEs:

$$\frac{dx_1}{dt} = 2x_1 - 3x_2$$
$$\frac{dx_2}{dt} = x_2 - 2x_1,$$

   subject to $x_1(0) = 8$, $x_2(0) = 3$.

6. Implement one step of the steepest descent method to find the minimum of the function $f(x_1, x_2) = 8x_1^2 + 4x_1x_2 + 5x_2^2$ starting at the point $(10, 10)$.