Quiz 1, solns

1) $e^{-2\ln x} = x^{-2}$

2) $\left(\sqrt{x} + \dfrac{x+1}{x^2+1}\right)' = \dfrac{1}{2\sqrt{x}} + \dfrac{x^2+1 - (x+1)2x}{(x^2+1)^2}$

3) $\displaystyle\int \dfrac{dx}{x^2 - 9}$

$\dfrac{1}{x^2-9} = \dfrac{1}{(x+3)(x-3)} = \dfrac{A}{x+3} + \dfrac{B}{x-3}$

$\begin{array}{l} x^1: 0 = A+B \\ x^0: 1 = -3A + 3B \end{array} \Bigg\} \begin{array}{l} A = -\dfrac{1}{6} \\ B = \dfrac{1}{6} \end{array}$

$-\dfrac{1}{6}\displaystyle\int \dfrac{dx}{x+3} + \dfrac{1}{6}\displaystyle\int \dfrac{dx}{x-3} = -\dfrac{1}{6}\ln|x+3| + \dfrac{1}{6}\ln|x-3|$

4) $h = f \circ g = f(g(\vec{x})) = f\begin{pmatrix} Ax_1 + Bx_2 \\ Cx_1 + Dx_2 \end{pmatrix} = \begin{pmatrix} (aA+bC)x_1 + (aB+bD)x_2 \\ (cA+dC)x_1 + (cB+dD)x_2 \end{pmatrix}$

$H = F \cdot G = \begin{bmatrix} aA + bC & aB + bD \\ cA + dC & cB + dD \end{bmatrix}$

5) $[A \,|\, p] = \begin{bmatrix} 1 & 2 & -3 & 4 & | & 1 \\ 4 & 8 & 12 & -8 & | & 2 \\ 2 & 3 & 2 & 1 & | & 3 \\ -3 & -1 & 1 & -4 & | & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 8 & 12 & -8 & | & 2 \\ 1 & 2 & -3 & 4 & | & 1 \\ 2 & 3 & 2 & 1 & | & 3 \\ -3 & -1 & 1 & -4 & | & 4 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 4 & 8 & 12 & -8 & | & 2 \\ \boxed{1/4} & 0 & -6 & 6 & | & 1 \\ \boxed{1/2} & -1 & -4 & 5 & | & 3 \\ \boxed{-3/4} & 5 & 10 & -10 & | & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 8 & 12 & -8 & | & 2 \\ \boxed{-3/4} & 5 & 10 & -10 & | & 4 \\ \boxed{1/2} & -1 & -4 & 5 & | & 3 \\ \boxed{1/4} & 0 & -6 & 6 & | & 1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 4 & 8 & 12 & -8 & | & 2 \\ \boxed{-3/4} & 5 & 10 & -10 & | & 4 \\ \boxed{1/2} & \boxed{-1/5} & -2 & 3 & | & 3 \\ \boxed{1/4} & \boxed{0} & -6 & 6 & | & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 8 & 12 & -8 & | & 2 \\ \boxed{-3/4} & 5 & 10 & -10 & | & 4 \\ \boxed{1/4} & \boxed{0} & -6 & 6 & | & 1 \\ \boxed{1/2} & \boxed{-1/5} & -2 & 3 & | & 3 \end{bmatrix} \rightarrow$

$$\Rightarrow \begin{bmatrix} 4 & 8 & 12 & -8 & | & 2 \\ -3/4 & 5 & 10 & -10 & | & 4 \\ 1/4 & 0 & -6 & 6 & | & 1 \\ 1/2 & -1/5 & 1/3 & 1 & | & 3 \end{bmatrix}$$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3/4 & 1 & 0 & 0 \\ 1/4 & 0 & 1 & 0 \\ 1/2 & -1/5 & 1/3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 4 & 8 & 12 & -8 \\ 0 & 5 & 10 & -10 \\ 0 & 0 & -6 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$