

Quiz 1, solns

$$1) e^{-2 \ln x} = x^{-2}$$

$$2) \left(\sqrt{x} + \frac{x+1}{x^2+1} \right)' = \frac{1}{2\sqrt{x}} + \frac{x^2+1 - (x+1)2x}{(x^2+1)^2}$$

$$3) \int \frac{dx}{x^2-9}$$

$$\frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)} = \frac{A}{x+3} + \frac{B}{x-3}$$

$$\begin{cases} x^1: 0 = A+B \\ x^0: 1 = -3A+3B \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{6} \\ B = \frac{1}{6} \end{cases}$$

$$-\frac{1}{6} \int \frac{dx}{x+3} + \frac{1}{6} \int \frac{dx}{x-3} = -\frac{1}{6} \ln|x+3| + \frac{1}{6} \ln|x-3|$$

$$4) h = f \circ g = f(g(\vec{x})) = f \begin{pmatrix} Ax_1 + Bx_2 \\ Cx_1 + Dx_2 \end{pmatrix} = \begin{pmatrix} (aA+bC)x_1 + (aB+bD)x_2 \\ (cA+dC)x_1 + (cB+dD)x_2 \end{pmatrix}$$
$$H = F \cdot G = \begin{bmatrix} aA+bC & aB+bD \\ cA+dC & cB+dD \end{bmatrix}$$

$$5) [A|p] = \left[\begin{array}{cccc|c} 1 & 2 & -3 & 4 & 1 \\ 4 & 8 & 12 & -8 & 2 \\ 2 & 3 & 2 & 1 & 3 \\ -3 & -1 & 1 & -4 & 4 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 4 & 8 & 12 & -8 & 2 \\ 1 & 2 & -3 & 4 & 1 \\ 2 & 3 & 2 & 1 & 3 \\ -3 & -1 & 1 & -4 & 4 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 4 & 8 & 12 & -8 & 2 \\ \textcircled{1/4} & 0 & -6 & 6 & 1 \\ \textcircled{1/2} & -1 & -4 & 5 & 3 \\ \textcircled{-3/4} & 5 & 10 & -10 & 4 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 4 & 8 & 12 & -8 & 2 \\ \textcircled{-3/4} & 5 & 10 & -10 & 4 \\ \textcircled{1/2} & -1 & -4 & 5 & 3 \\ \textcircled{1/4} & 0 & -6 & 6 & 1 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 4 & 8 & 12 & -8 & 2 \\ \textcircled{-3/4} & 5 & 10 & -10 & 4 \\ \textcircled{1/2} & \textcircled{-1/5} & -2 & 3 & 3 \\ \textcircled{1/4} & 0 & -6 & 6 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cccc|c} 4 & 8 & 12 & -8 & 2 \\ \textcircled{-3/4} & 5 & 10 & -10 & 4 \\ \textcircled{1/4} & 0 & -6 & 6 & 1 \\ \textcircled{1/2} & \textcircled{-1/5} & -2 & 3 & 3 \end{array} \right] \rightarrow$$

$$\rightarrow \left[\begin{array}{cccc|c} 4 & 8 & 12 & -8 & 2 \\ -3/4 & 5 & 10 & -10 & 4 \\ 1/4 & 0 & -6 & 6 & 1 \\ 1/2 & -1/5 & 1/3 & 1 & 3 \end{array} \right]$$

$$\Rightarrow L = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ -3/4 & 1 & 0 & 0 & 4 \\ 1/4 & 0 & 1 & 0 & 1 \\ 1/2 & -1/5 & 1/3 & 1 & 3 \end{array} \right]$$

$$U = \left[\begin{array}{cccc|c} 4 & 8 & 12 & -8 & 2 \\ 0 & 5 & 10 & -10 & 4 \\ 0 & 0 & -6 & 6 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array} \right]$$

$$P = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array} \right]$$

$$2x + 10y$$

$$s - x = 2s - 2 \quad (1)$$

$$\left(\frac{1+x}{1+s} + x \right) \quad (S)$$

$$\left. \begin{array}{l} \frac{2b}{s-x} \\ P \rightarrow s-x \end{array} \right\} \quad (2)$$

$$\frac{B}{s-x} + \frac{A}{s+x} = \frac{1}{(s-x)(s+x)} = \frac{1}{s^2 - x^2}$$

$$\begin{cases} B + A = 0 & \text{if } x \\ 8B + 4A = 1 & \text{if } x^2 \end{cases}$$

$$\frac{1}{s-x} = \frac{xb}{s-x} \left[\frac{1}{s} + \frac{xb}{s+x} \right] \frac{1}{s}$$

$$H = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$H = F \cdot G = H$$

$$\left[\begin{array}{cccc|c} 1 & 8 & 15 & 8 & 4 \\ 5 & 2 & 1 & 5 & 1 \\ 2 & 1 & 1 & -2 & -3 \end{array} \right] = [A|b] \quad (2)$$

$$\left[\begin{array}{cccc|c} 5 & 2 & 15 & 8 & 4 \\ 1 & 2 & 2 & 2 & 1 \\ 2 & 1 & 1 & -2 & -3 \end{array} \right] \rightarrow$$

$$\left[\begin{array}{cccc|c} 5 & 2 & 15 & 8 & 4 \\ 1 & 2 & 2 & 2 & 1 \\ 2 & 1 & 1 & -2 & -3 \end{array} \right] \rightarrow$$

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