ECH3301 Process Analysis and Design 11/08/10

**Exam 2 Name** _____

*This is a closed book exam. Calculators are allowed. You can use one 8.5×11 sheet of paper (both sides) with notes of your choice.*

1. Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. Assume that the rate at which the virus spreads is proportional not only to the number $x$ of infected students but also to the number of students not infected, i.e. the rate is proportional to the product of the two numbers. Determine the number of infected students afters 6 days, if it is further observed that after 4 days $x(4) = 50$.

2. A water tank, in the shape of a conical funnel, with is apex at the bottom and vertical axis, is 12 ft across the top and 18 ft high. It has a hole at its apex of radius 1 in. When will the tank be empty if initially it is filled with water?

   *[Hint: When water flows from a tank through a small hole in its bottom, it has been proved that*

   $$\frac{dV}{dt} = -ka\sqrt{h},$$

   *where $V$ is the number of cubic feet of water in the tank at time $t$ sec, $k$ is a proportionality constant (=4.8), $a$ is the area of the hole in square feet and $h$ is the height in feet of the water above the hole at time $t$. You'll also need a relationship between the $V$ and the cross-sectional area of the water surface at time $t$. ]*

3. Solve the equation of the undamped forced motion with a sinusoidal impressed force, i.e.

   $$\frac{d^2x}{dt^2} + \omega^2 x = F_0 \sin \gamma t, \qquad x(0) = 0, \qquad x'(0) = 0,$$

   where $F_0$ is a constant and $\gamma \neq \omega$. How would the solution change if $\gamma \to \omega$ (known as pure resonance)?

4. Using Laplace transforms, solve the system

   $$\begin{aligned} x'' + y' + 3x &= 15e^{-t} \\ y'' - 4x' + 3y &= 15 \sin 2t, \end{aligned}$$

   subject to

   $$\begin{aligned} x(0) = 35, \quad x'(0) = -48, \\ y(0) = 27, \quad y'(0) = -55, \end{aligned}$$