ECH3301 Process Analysis and Design Fall 2010

HW 1 Calculus Review Due August 30

1. Evaluate the following limits:

   a)
   $$\lim_{x\to\infty}\left(e^x \operatorname{arccot} x\right)$$
   [Hint: Convert the indeterminate form of type $(0\times\infty)$ to $(0/0)$ and use L'Hospital's rule.]

   b)
   $$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\ln(1+x)}\right)$$

2. Compute the following integrals without using tables of integrals. Show all the steps.

   a)
   $$\int e^{2x}\cos x\,dx$$

   b)
   $$\int \sqrt{1-t^2}\,dt$$
   [Hint: Use trigonometric substitution.]

   c)
   $$\int \frac{x}{x^3-1}\,dx$$
   [Hint: Use partial fractions. If you encounter $\int \frac{Ax+B}{ax^2+bx+c}\,dx$, write it as a sum of two easier integrals, $U\int \frac{(ax^2+bx+c)'}{ax^2+bx+c}\,dx + V\int \frac{1}{ax^2+bx+c}\,dx$ and equate the left-hand side and right-hand side powers of x in the numerators to determine constants U and V.]

3. Find the moment of inertia of a hemispherical surface about the x-axis, assuming the surface to be a homogeneous lamina (thin sheet) of mass M.

   [Hint: The moment of inertia is $I=\iint\limits_S \sigma(y^2+z^2)\,dS$, where σ is the constant surface density. Remember that once we have a surface parametrization $\mathbf{r}(x,y)$, we can express the surface integral as ordinary double integral, $I=\int\limits_S f\,dS=\iint\limits_R f(\mathbf{r}(x,y))\left|\frac{\partial\mathbf{r}}{\partial x}\times\frac{\partial\mathbf{r}}{\partial y}\right|\,dx\,dy$, with dR being a projection of dS into $dx\,dy$. Use polar coordinates and tables of integrals (if needed) to evaluate the integral.]