ECH3301 Process Analysis and Design — Fall 2010

HW 3 Orthogonality — Due September 22

1. **(Least squares)** After studying a certain type of cancer, a researcher hypothesizes that in the short run the number (y) of malignant cells in a particular tissue grows exponentially with time (t). That is, $y = \alpha_0 e^{\alpha_1 t}$. Determine least squares estimates for the parameters α_0 and α_1 from the researcher's observed data given below.

t (days)	1	2	3	4	5
y (cells)	16	27	45	74	122

[Hint: To be able to use linear least squares, we need the residuals to be linear in all unknowns. What common transformation converts an exponential function into a linear function?]

2. **(Fourier expansion)** Using the standard inner product, determine the Fourier expansion of x with respect to the basis B, where

$$x = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \text{ and } B = \left\{ \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \frac{1}{\sqrt{6}} \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix} \right\}.$$

[Hint: Recall that any vector b can be resolved into n mutually orthogonal basis vectors $q_1, q_2, \ldots q_n$: $b = x_1 q_1 + x_2 q_2 + \cdots + x_n q_n$. To determine Fourier coefficient x_1, multiply both sides by q_1^T. Proceed by analogy with other Fourier coefficients x_i.]

3. **(QR factorization)** Let

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & -3 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

   a) Determine the rectangular QR factorization of A.

   b) Use the QR factors to determine the least squares solution to $Ax = b$.

   [Hint: To apply QR factorization to least squares, substitute $A = QR$ into the least squares equation $A^T Ax = A^T b$.]