ECH3301 Process Analysis and Design 9/27/10

Exam 1, v.2 Name ______________________________

This is a closed book exam. No electronics allowed. You can use one 8.5×11 sheet of paper (both sides) with notes of your choice. Choose between problems no.1 and 2. If you start both, the higher score of the two will be used in the final average.

1. The CT scanners examine the patient from different directions and produce a matrix giving the densities of bone and tissue at each point. Mathematically, the problem is to recover a matrix from its projections. In the 2 by 2 case, can you recover the matrix A if you know the sum along each row and down each column?

2. Consider a particular species of wildflower in which each plant has several stems, leaves and flowers, and for each plant let the following holds. S = the average stem length (in inches). L = the average leaf width (in inches). F = the number of flowers. Four particular plants are examined, and the information is tabulated in the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 10 \\ 2 & 1 & 12 \\ 2 & 2 & 15 \\ 3 & 2 & 17 \end{bmatrix}.$$

   For these four plants, determine whether or not there exists a linear relationship between S, L and F. In other words, do there exist constants α_0, α_1, α_2, and α_3 such that $\alpha_0 + \alpha_1 S + \alpha_2 L + \alpha_3 F = 0$?

3. A small company has been in business for three years and has recorded annual profits (in thousands of dollars) as follows:

Year	1	2	3
Sales	7	4	3

   Assuming that there is a linear trend in the declining profits, predict the year and the month in which the company begins to lose money.

4. Determine the eigenvalues and eigenvectors of the following matrix

$$A = \begin{bmatrix} 0 & 6 & 3 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

   Is the matrix deficient in eigenvectors in the sense that there fails to exist a complete linearly independent set?

   [Hint: When solving a cubic equation, note that if the coefficients α_i in $\lambda^n + \alpha_{n-1}\lambda^{n-1} + \cdots + \alpha_1\lambda + \alpha_0 = 0$ are integers, then every integer solution is a factor of α_0. Evaluate $p(\lambda)$ for each λ that satisfies this condition to find one or more eigenvalues. To find the remaining ones, use long division.]