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ECH3301 Process Analysis and Design  
HW 4 1st Order ODEs

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Fall 2010  
Due Oct 13

1. **(Dilution problem)** A large tank is filled to capacity with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well-mixed solution is pumped out at the rate of 10 gal/min. Determine a differential equation for the amount  $A(t)$  of salt in the tank at time  $t$ . Find the salt content in the mixture as a function of time. When is the tank empty?
2. **(Directional field)** Sketch, by hand or using a computer, the directional field and an approximate solution curve (integral curve) that passes through each of the indicated points.

$$\frac{dy}{dx} = x^2 - y^2$$

- a)  $y(-2) = 1$
- b)  $y(3) = 0$
- c)  $y(0) = 2$
- d)  $y(0) = 0$

Use a square lattice with  $-3 \leq x, y \leq 3$ .

3. **(Linear DE)** Find the general solution of the given differential equation. Give the largest interval over which the general solution is defined.

$$x^2 y' + x(x+2)y = e^x$$

3. **(Exact DE)** Solve the given initial-value problem (IVP) by finding an appropriate integrating factor.

$$x dx + (x^2 y + 4y) dy = 0, \quad y(4) = 0.$$

4. **(Substitution)** Solve the following differential equations by using an appropriate substitution.

a)

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

b)

$$\frac{dy}{dx} = y(xy^3 - 1)$$