HW 3, solns

1) $y = \alpha_0 \cdot e^{\alpha_1 t}$ (non-linear)

$$\underbrace{\ln y}_{b} = \underbrace{\ln \alpha_0}_{\alpha_2} + \alpha_1 t$$

$\alpha_2 + \alpha_1 t = b$ (linear)

$\downarrow$ Matrix form

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} \alpha_2 \\ \alpha_1 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}}_{b} \iff Ax = b$$

$$A^T A x = A^T b$$

$$\bar{x} = (A^T A)^{-1} A^T b$$

$$A^T A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 55 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{5 \cdot 55 - 15 \cdot 15} \cdot \begin{bmatrix} 55 & -15 \\ -15 & 5 \end{bmatrix} = \begin{bmatrix} 1.1 & -0.3 \\ -0.3 & 0.1 \end{bmatrix}$$

$$\Rightarrow \bar{x} = \begin{bmatrix} 2.275 \\ 0.507 \end{bmatrix} \Rightarrow \alpha_1 = x_2 = \underline{\underline{0.507}}$$

$$\alpha_2 = e^{x_1} = \underline{\underline{9.731}}$$

2) $x = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$, q_1, q_2, q_3 (given)

$$x = \alpha_1 q_1 + \alpha_2 q_2 + \alpha_3 q_3$$

$$\alpha_1 = q_1^T x = \frac{1}{\sqrt{2}}$$

$$\alpha_2 = q_2^T x = -\frac{1}{\sqrt{3}}$$

$$\alpha_3 = q_3^T x = -\frac{5}{\sqrt{6}}$$

$$x = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + -\frac{1}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \frac{5}{6}\begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

3) a) $Q = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 0 & 1/\sqrt{3} & 0 \end{bmatrix}$

$$R = \begin{bmatrix} \sqrt{3} & \sqrt{3} & -\sqrt{3} \\ 0 & \sqrt{3} & \sqrt{3} \\ 0 & 0 & \sqrt{6} \end{bmatrix}$$

b) Solve $Rx = Q^T b$ to get

$$x = \begin{bmatrix} 2/3 \\ 1/3 \\ 0 \end{bmatrix}.$$