
ECH3301 Process Analysis and Design Spring 2012

Rec 3 $M \times N$ systems

1. (**$M \times N$ system**) Determine the general solution of the following homogeneous system:

$$\begin{aligned}x_1 + 2x_2 + x_3 + 2x_4 &= 0, \\2x_1 + 4x_2 + x_3 + 3x_4 &= 0, \\3x_1 + 6x_2 + x_3 + 4x_4 &= 0.\end{aligned}$$

2. (**Inverse matrix**) Find the matrix X such that $X = AX + B$, where

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 3 \end{bmatrix}.$$

3. (**Linear independence**) Is the following set of vectors linearly independent? If not, write one of the vectors as a linear combinations of others.

$$\{(1 \ 2 \ 3), (0 \ 4 \ 5), (0 \ 0 \ 6), (1 \ 1 \ 1)\}.$$

[Hint: To write one vector as a combination of others in a dependent set, place the vectors as columns in A and find the echelon form. This reveals the dependence relationships among columns of A .]

4. (**$N \times N$ system**) In order to grow a certain crop, it is recommended that each square foot of ground be treated with 10 units of P, 9 units of K and 19 units of N. Suppose that there are three brands of fertilizer on the market, say X, Y, Z. One pound of X contains 2 units of P, 3 units of K and 5 units of N. One pound of Y contains 1 unit of P, 3 units of K and 4 units of N. One pound of Z contains only 1 unit of P and 1 unit of N. Take into account the obvious fact that a negative number of pounds of any brand can never be applied, and suppose that because of the way fertilizer is sold only an integral number of pounds of each brand will be applied.

Under these constraints, what are all the combinations of the three brands that can be applied to satisfy the recommendations exactly?

5. (**DC Electrical Circuit**)

Suppose that $R_i = i$ ohms and $E_i = i$ volts in the circuit below. Determine the six indicated currents.

