Modelling Extreme Field Discharge on Overlapping Grids

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Objectives

To develop a self-contained software framework for the simulation of a wide class of cold electrical discharges at atmospheric pressure, including positive streamers.

- A fluid model accounting for photoionization and electron energy equation is solved efficiently on overlapping, structured grids.
- The governing equations are discretized with finite-difference (FDM) and finite-volume (FVM) operators and transformed into parametric space to ease the implementation of high-resolution conservative schemes on curvilinear grids.
- Collocated grid arrangement further increases efficiency of the implementation.
- Operator splitting is used to include additional source terms, including those due to chemical reactions and/or the axi-symmetry of the domain.

Composite Grids

- Structured, overlapping grids are efficient in terms of computer time and memory while allowing meshing of complicated and/or moving geometries.
- We combined boundary-fitted curvilinear grids near boundaries with background Cartesian grids in the interior.
- Component grids are logically rectangular, defined by smooth mappings from parameter space $\boldsymbol{r}$ (unit square, cube) to physical space $\boldsymbol{x}$:

$$\boldsymbol{x} = \boldsymbol{G}(\boldsymbol{r}), \quad \boldsymbol{r} \in [0,1] \times [0,1], \quad \boldsymbol{x} \in \mathbb{R}^2.$$

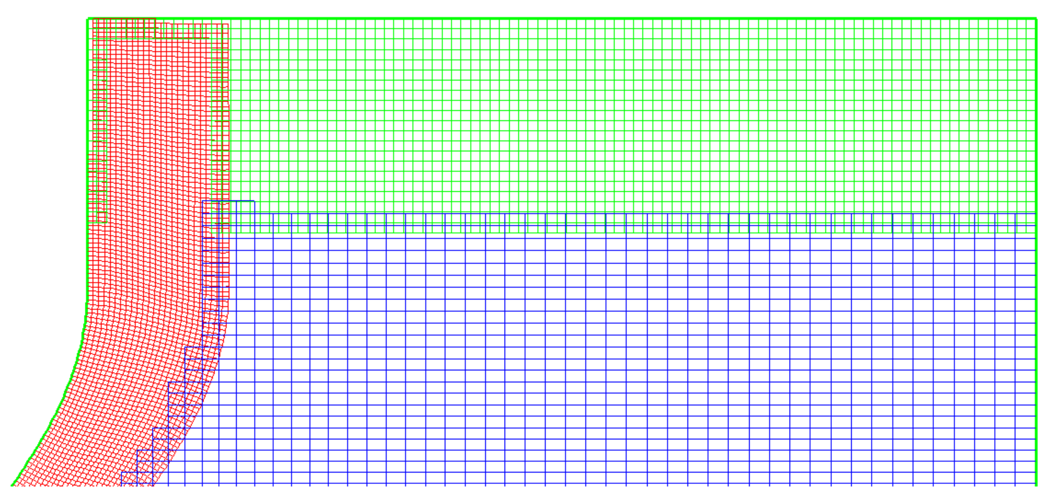

Figure 1: Section of anode neighborhood (red grid) in point-to-plane

Model Description

- Plasma chemistry: minimal, nitrogen and air.
- Transport parameters (LFA):
  1. Analytical formulas (literature), or
  2. Look-up tables (Bolsig+) with cubic spline interpolation.

Electric Field: FDM

The electric potential ϕ is governed by

$$\Delta \phi = -\frac{\rho}{\epsilon_0}, \qquad (1)$$

with electric field $\boldsymbol{E} = -\nabla\phi$ and space charge density $\rho = \sum_j q_j n_j$.

- 2nd order FDM on vertex-centered grid;
- Banded algebraic system: direct/iterative/PETSC solvers;
- Coordinate singularity: L'Hopital's rule;
- Dielectrics: surface charge accumulation, sub-domain iterations;
- Semi-implicit correction to remove dielectric relaxation time scale.

Species Densities: FVM

For a smooth grid mapping from a Cartesian to physical space we transform to Cartesian space, obtaining

$$\frac{\partial n}{\partial t} + \frac{1}{J}\frac{\partial}{\partial r_1}\hat{f}_1 + \frac{1}{J}\frac{\partial}{\partial r_2}\hat{f}_2 = \hat{R}. \qquad (2)$$

For cell average N over a vertex-centered grid cell $\boldsymbol{i} = (i,j)$ at time t_n,

$$N_{\boldsymbol{i}}^n = \frac{1}{\Delta x \Delta y}\int_{y_{j-1/2}}^{y_{j+1/2}}\int_{x_{i-1/2}}^{x_{i+1/2}} n(x,y,t_n)\,dxdy,$$

we apply 1st order operator splitting to flux terms S_F and source terms S_R, so that $N_{\boldsymbol{i}}^{n+1} = S_F(\Delta t) S_R(\Delta t) N_{\boldsymbol{i}}^n$, where

$$N_{\boldsymbol{i}}^{n+1} = S_R(\Delta t) N_{\boldsymbol{i}}^*,\; N_{\boldsymbol{i}}^* = S_F(\Delta t) N_{\boldsymbol{i}}^n.$$

S_F represents fully discrete flux-differencing, thus $N_{\boldsymbol{i}}^* =$

$$N_{\boldsymbol{i}}^n - \frac{\Delta t}{J_{\boldsymbol{i}}}\left[\frac{\hat{F}_{1,i+1/2,j} - \hat{F}_{1,i-1/2,j}}{\Delta x} + \frac{\hat{F}_{2,i,j+1/2} - \hat{F}_{2,i,j-1/2}}{\Delta y}\right],$$

with flux functions $\hat{F}$ given by an upwind/high-resolution method. S_R is implemented as an explicit Euler.

Extreme field, nano-second discharges in complicated geometries can be efficiently analyzed on HPC cluster using structured, composite grids with the combination of finite difference and finite volume methods.

Photoionization

System of n Helmholtz equations gives S_{ph} source

$$\left(-\nabla^2 + \lambda_j^2\right) S_{ph,j} = S_i, \qquad S_{ph} = f_q \sum_{j=1}^{n} A_j S_{ph,j} \quad (3)$$

with quenching factor f_q and emission intensity $\propto$ ionization source $S_i = \sum_r^{n_r} \nu_{i,r} n_e$, where $r = 1 \ldots n_r$.

- Equivalent to generalized Eddington approximations of the radiative transfer equation (e.g. Eddington and SP3 models);
- Derived from Zheleznyak integral model by fitting absorption function by n exponential ($\rightarrow \lambda_j$) and interpreting integral photoionization rate as the appropriate Green's function for the corresponding differential model.

Results: Point-to-plane in air

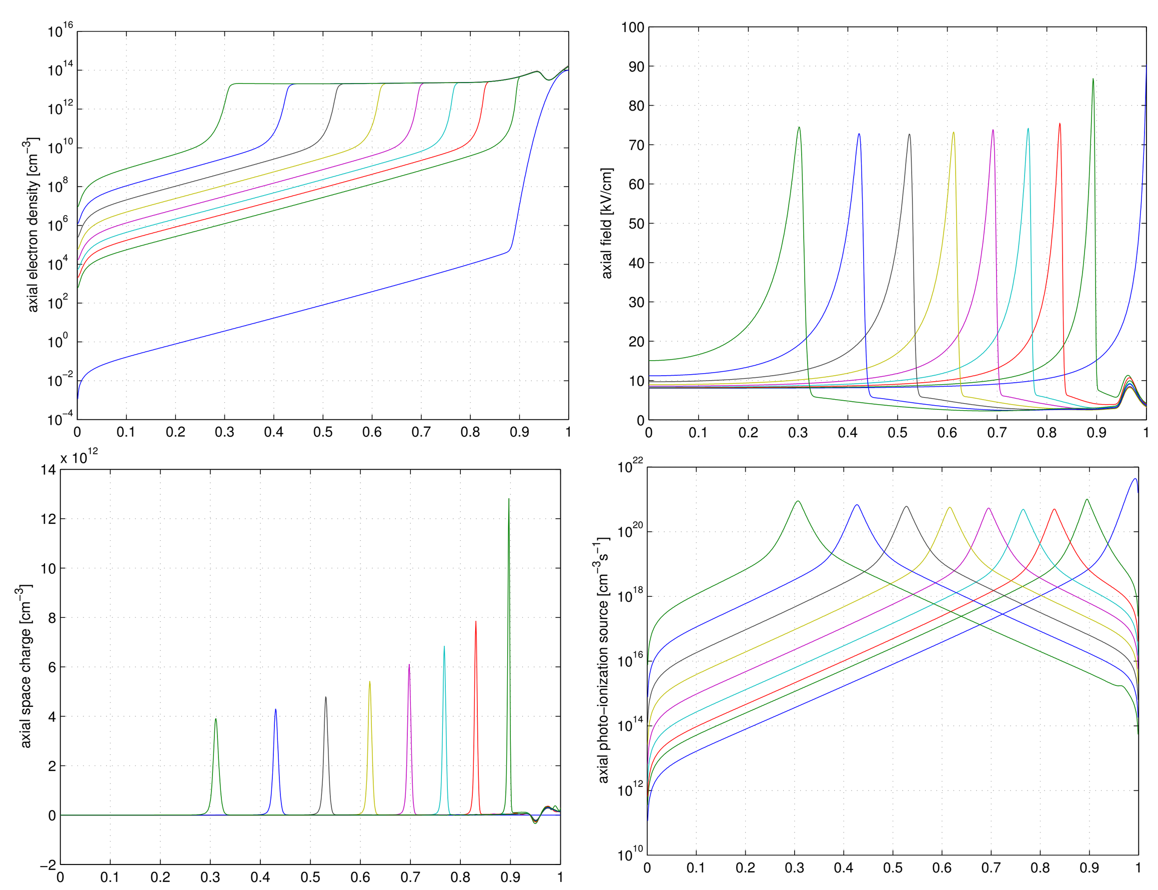

Figure 2: Electron density, electric field, space charge and photoion.

Results, continued

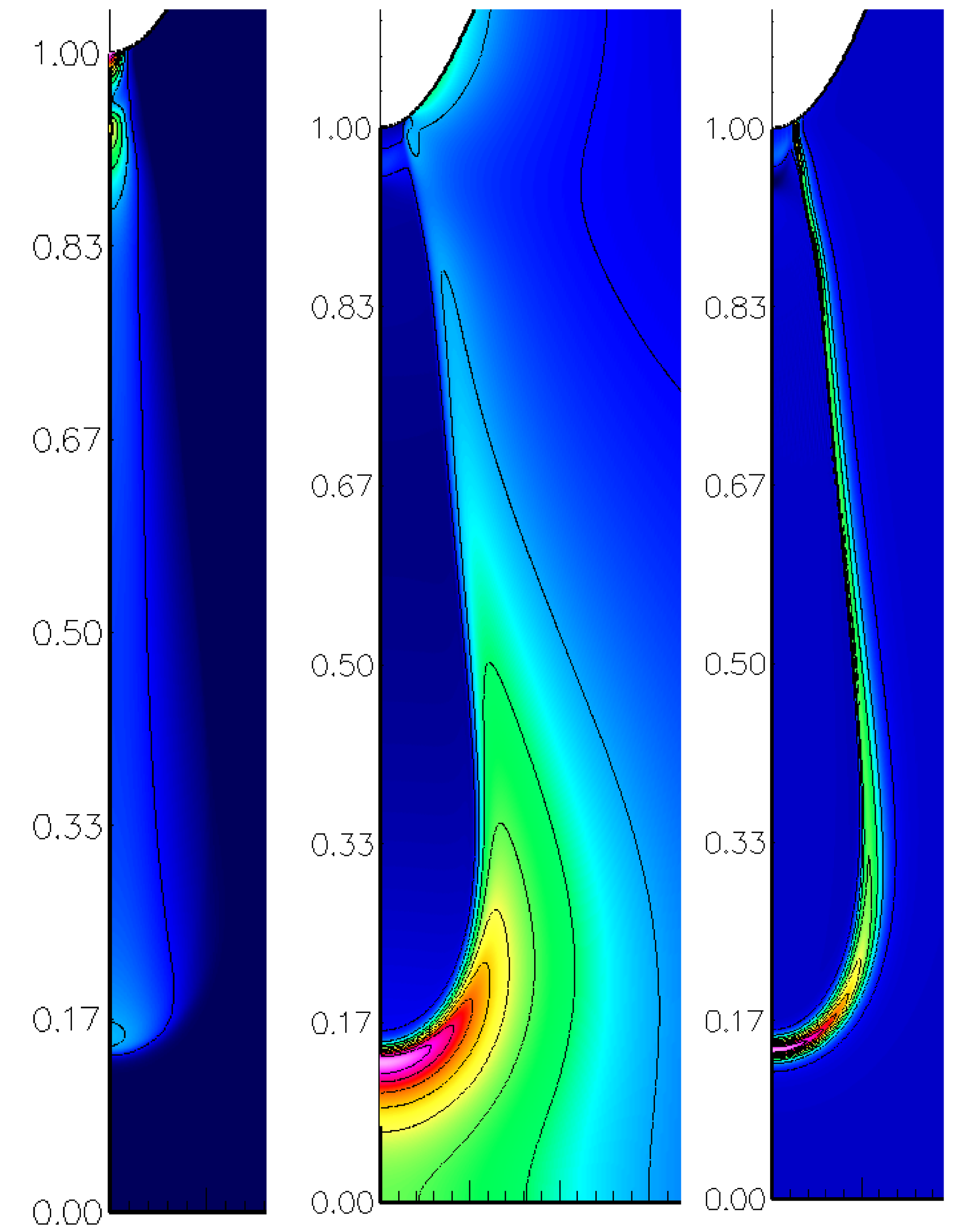

Figure 3: Electron density, electric field and space charge density in a cathode-directed streamer (gap=1cm, V=11kV, tip radius 500 μm

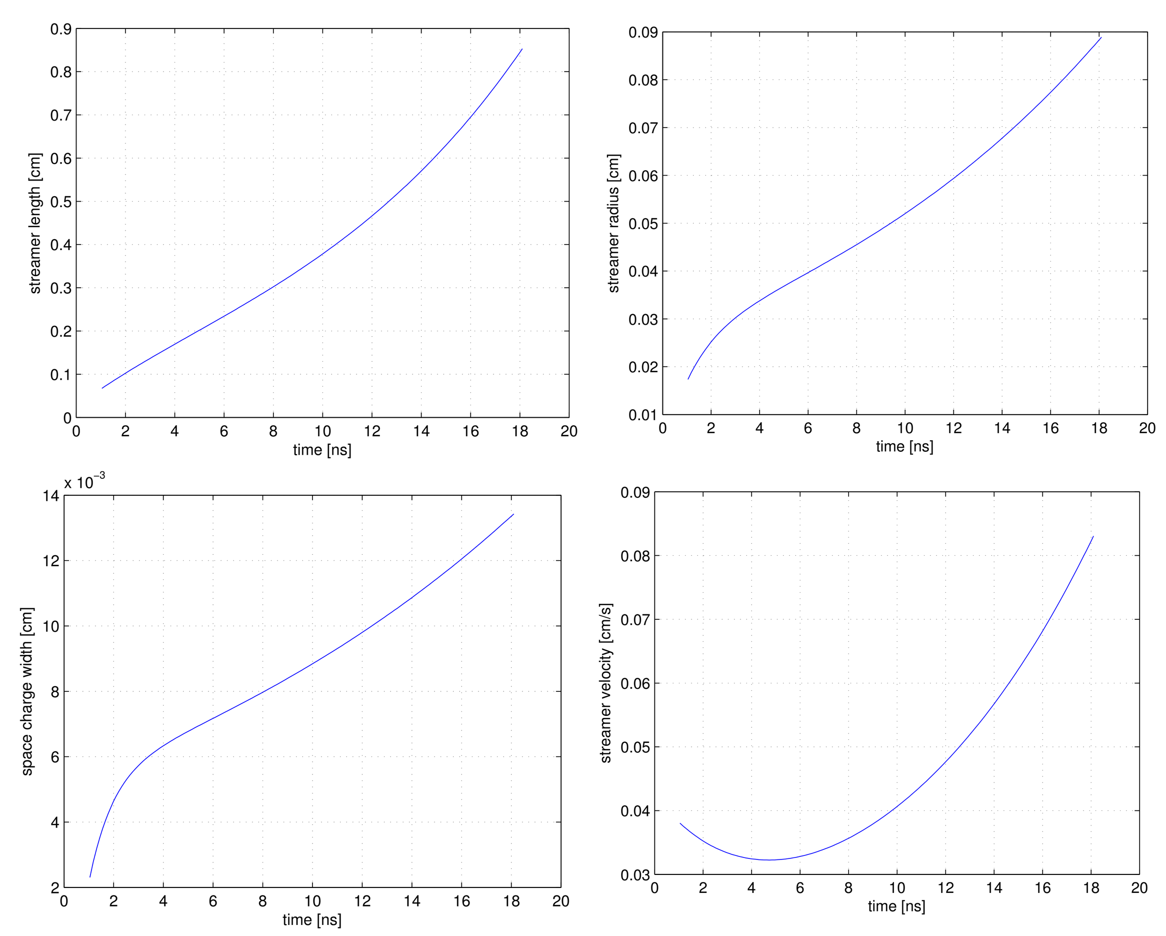

Figure 4: Streamer length, radius, space charge width and velocity

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