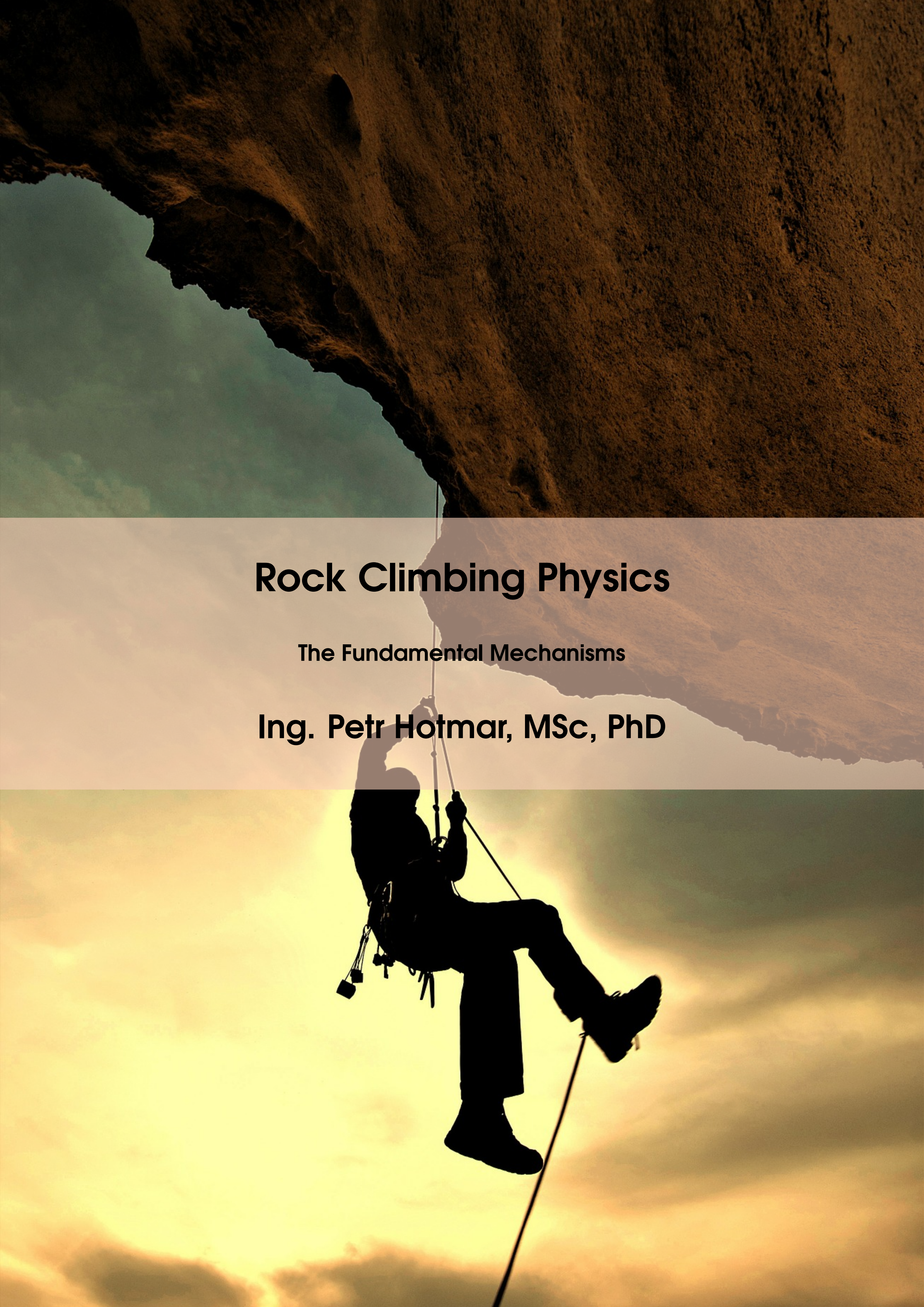
Rock Climbing Physics

The Fundamental Mechanisms

Ing. Petr Hotmar, MSc, PhD



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1. Introduction

1.1 Foreword

This is a no-nonsense treatise on scientific aspects of rock climbing, as well as technical discussion on some advanced climbing techniques. The mathematical and physical principles discussed herein aim to provide a self-consistent body of information from which practical advice on climbing techniques, use of equipment and safety, can be deduced by an astute reader. Thus, rather than providing specific rock climbing instructions, as typical textbooks do, we encourage the reader to grasp the underlying mechanisms to be able to improvise practical solutions to any given situation. The second objective of this text is to dispel some commonly re-curing myths, which tend to arise from misleading scientific information, often presented in the literature and/or promulgated in the community.

As a person with experience in natural sciences and rock climbing I have used knowledge of fundamental physical principles especially in challenging situations, such as roped solo leading in alpine conditions. It is my belief that you too can use the knowledge of relevant natural phenomena to derive useful, even life-saving, guidance in situations where unique technical and mental challenges, coupled with an urgent need for improvisation, leave no room for error. Being able to successfully and efficiently solve these problems of the vertical world, though, tends to reward us with the most valuable, satisfying and unforgettable experiences.

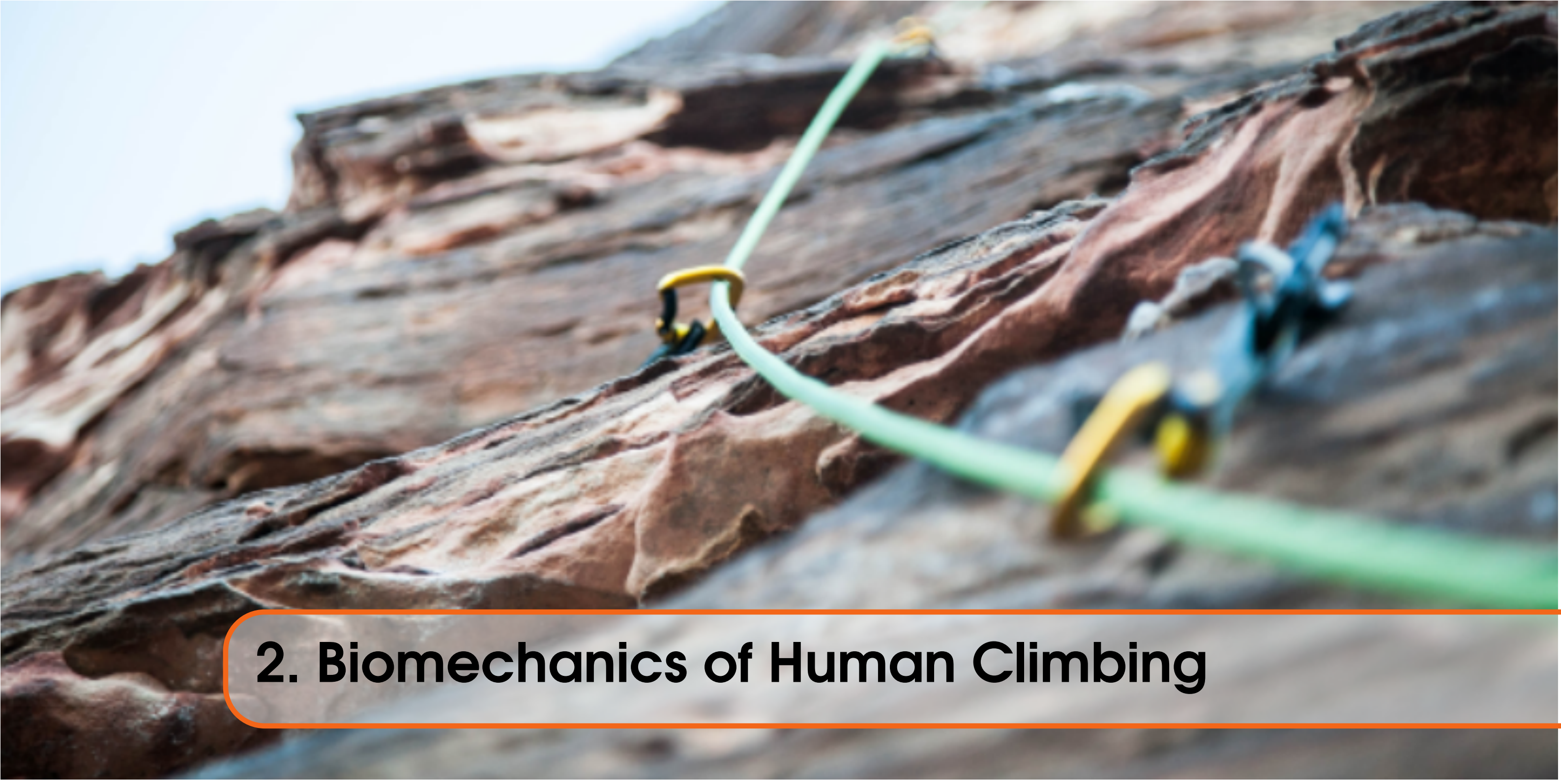
Today there is extensive literature available covering the basic climbing methodology and related aspects. This book is intended to complement and expand such instructions, and is thus hoped to be of use primarily to the scientifically curious and/or climbers with prior experience. Any constructive criticism, comments or suggestions are always welcome.

1.2 About Author

Petr Hotmar is a scientist and rock climber.

He received Master of Science in Cybernetics from The Institute of Chemical Technology, Prague, Czech Republic, Master of Science and Ph.D. in Chemical and Biomedical Engineering from Florida State University, Tallahassee, USA, and a post-doctoral fellowship in Plasma Physics from LAPLACE Laboratory in Toulouse, France. He specializes in computational physics, applied mathematics and medical biology.

He started rock climbing under the tutelage of a rock climbing legend Jaroslav Zivor, and later joined the High Mountain Sports rock climbing club in Brno, Czech Republic. He enjoys roped soloing and multi-pitch climbing in Moravian Karst region in the Czech Republic, High Tatra mountains in Slovakia, and the Alps.



2. Biomechanics of Human Climbing

2.1 Human Biomechanics

During climbing, or any other human motion, the contractile units (skeletal muscles) act, typically through tension-resistant cable-like structures (tendons), on compression-resistant levers (bones) associated with fulcrum points (joints). Levers are commonly classified by the relative position of the fulcrum, effort (input force) and resistance (output force, load). Either the fulcrum, resistance or effort is in the middle, corresponding to, respectively, classes 1, 2 and 3, see Fig. In class 1 lever (e.g. seesaw or atlanto-occipital joint) the mechanical advantage (MA, see Eq. 4.2 for definition) is arbitrary – the action may change the speed of movement and/or amount of force transmitted. In class 2 lever (e.g. wheelbarrow or ankle joint) the force is magnified ($MA > 1$), decreasing the effort, at the expense of speed and distance, e.g. allowing the calf muscles to lift weight larger than its tensile strength. In class 3 lever (e.g. hammer or elbow joint) the speed is multiplied at the expense of force ($MA < 1$) and distance traveled, e.g. allowing a small contraction of the biceps muscle to translate into a large range of motion of the forearm.

The fundamental principles of human biomechanics that are applicable to rock climbing are based on Newton's laws of motion, conservation of linear/angular momentum and mechanical energy, and force and torque static equilibria. In the first approximation the motion can be analyzed by reducing the human body or its segments into a collection of point masses/particles with the mass lumped at the geometric center of the segment. In second approximation, used to develop the majority of the models, we allow for the body mass to be distributed continuously throughout rather than concentrated at a particle point, representing the body by a series of interconnected rigid (no deformation) rods. The most precise models (typically beyond the needs of rock-climbing applications) allow for the deformation of the body segments, e.g. by representing them as interconnected springs.

Exercise 2.1 Assume a climber of mass m is hanging by his hands (hand distance d) from an overhanging rock (force F acting on each hand), attempting to pull himself up (peak acceleration a). He fails to do so and, to relieve shoulder tension, he lets one hand go (assume radius of gyration k). What is the force exerted by the climber before and after he lets go his hand?

Solution: Writing Newton's 2nd law of motion we get

$$-mg + 2F = ma \implies F = \frac{1}{2}m(a+g), \tag{2.1}$$

where $g \sim 9.8$ m/s^2 is the average Earth's gravitational acceleration. We see that faster pull (larger acceleration) results in proportionally larger force and muscle fatigue. E.g. for $m = 80$ kg and $a = 5$ m/s^2 we get $F = 592$ N, which is equivalent to about 1.3 times the climber's weight.

Placing the body's center-of-mass at $d/2$, the conservation of angular momentum requires the torque $\tau = -Fd/2$ to satisfy

$$\tau = I\alpha, \tag{2.2}$$

where $I = mk^2$ is the moment of inertia and α is the angular acceleration gained by the climber after the hand slip.

The force balance after the hand slip is then

$$-mg + F = ma, \tag{2.3}$$

with the center-of-mass acceleration $a = \alpha d/2$. Solving for α and F from Eqs. 2.2 and 2.3 we obtain

$$\alpha = -cdg, \tag{2.4}$$
$$F = cIg, \tag{2.5}$$

where $c = 2/(d^2 + 2k^2)$.

We see that after the hand slip the force F on the remaining hand is inversely proportional to the square of the original distance between the hands, d. To maintain stable position and restore equilibrium in similar situations the climber has to optimally re-distribute the forces on the remaining supports. In the absence of supports the climber typically counteracts the body's tendency to rotate by shifting the center of mass, e.g. by placing a foot against the rock or flagging it in mid-air. If the angular acceleration is un-opposed, the climber's body swings can be analyzed, in the first approximation, by using a model of an elastic pendulum. ■

2.1.1 Climbing Technique

In general, the climber should maximize the efficiency (minimize energy expenditure) of each move by using correct technique. This typically involves

- Shifting the center-of-mass (COM) E.g. when making an extended arm reach the use of outside edge of the foot brings the ipsilateral hip, and thus the COM, closer to the wall (drop-knee technique).
- Use of opposing pressures (counter-pressure) between the limbs and/or fingers (e.g. stemming or gastons).
- Use of directional pressures to maximize traction (e.g. side pulls).
- Engagement of various muscle groups, including abdominal, which resists trunk rotation and thus increases energy efficiency of force transfers between the individual body segments.

The climber has to support its weight by appropriate use of tangential and normal forces acting at the and and footholds. Finding optimal body position and orientation can help minimize these contact forces and find balanced body positions with minimal muscle fatigue.

As when climbing the ladder, the legs should support the body weight while the hands contribute to the stability. The optimal re-distribution of forces on the foot/hand holds, along with postural adjustments, is the hallmark of an efficient rock-climbing move.

2.1.2 Mechanics of Musculoskeletal Tissues

In climbing the musculoskeletal system, especially the connective and muscle tissue, is subject to significant forces and stresses. Overuse and/or inefficient technique may lead to injury, which tends to heal slow. From an engineering standpoint, these tissues are often composite materials with nonhomogeneous and anisotropic properties. They are also viscoelastic, exhibiting both viscous (fluid-like) and elastic (solid-like) properties. In the first approximation described by a linear behavior, the following characteristics apply to an elastic material:

- time-independent behavior (e.g. shapre resumed instantaneously after load removed),
- modeled by springs which store potential energy,
- stress σ proportional to the strain ε, $\sigma = E\varepsilon$, where E is the elastic modulus,

and to the viscous material:

- time-dependent behavior (e.g. deformation and recovery is gradual),
- modeled by dashpots (piston-cylinder) which dissipate kinetic energy into heat by friction,
- stress σ proportional to the rate-of-strain $\dot{\varepsilon}$, $\sigma = \eta\dot{\varepsilon}$, where η is the dynamic viscosity and $\dot{\varepsilon} = \frac{d\varepsilon}{dt}$.

Similar to electrical circuits, the empirical models of viscoelastic tissues can be obtained by combining the springs and dashpots in series or parallel.

Seeing that the primary function of a muscle is to perform mechanical work by contraction, it must be assisted by multiple connective tissues:

- bones facilitate muscle actions,
- tendons (high tensile strength, rupture at about 60 MPa) transmit muscle-bone forces,
- ligaments (rupture at about 20 MPa) transmit bone-bone forces and stabilize the articulations (joints),
- cartilage facilitates articulations by increasing contact area and reducing friction.

The load-bearing ability of cartilage is particularly prominent in rock-climbing, as it is subjected to high tensile, shear and compressive stresses. E.g. the forces at the hip joint are several times the body weight during climbing, as the hip contact area is only a few square centimeters. Similarly, the forces supported by the tendons in the legs often exceed several times the body weight, especially during dynamic moves.

Exercise 2.2 Show how the force supported by the Achilles tendon during a dynamic climbing move ('dyno'), F_a, depends on the foot angle α at the time of push-off. Assume foot anatomy $l = (l_1, \dots l_4)$, foot weight m_f, and body weight m_b of a typical adult, and vertical acceleration during the push-off to be $a = g$.

Solution: The foot can be represented e.g. by a rigid-rod model depicted in Fig. 2.1, with Achilles tendon insertion point A, the fulcrum point B (talocrural joint) and point of flexion/extension D (metatarsophalangeal joint). A low foot mass allows to neglect inertial effects (pseudo-stationary approximation) and reduce the problem to that of static equilibrium.

The force balance yields

$$2F_g - W_b = m_b a, \tag{2.6}$$

with ground reaction force F_g and body weight $W_b = m_b g$.

The moment balance with respect to talocrural (ankle) joint yields

$$F_g\left(\frac{l_4}{2} + (l_2 + l_3)\cos\alpha\right) = F_a l + W_f l_2 \cos\alpha, \tag{2.7}$$

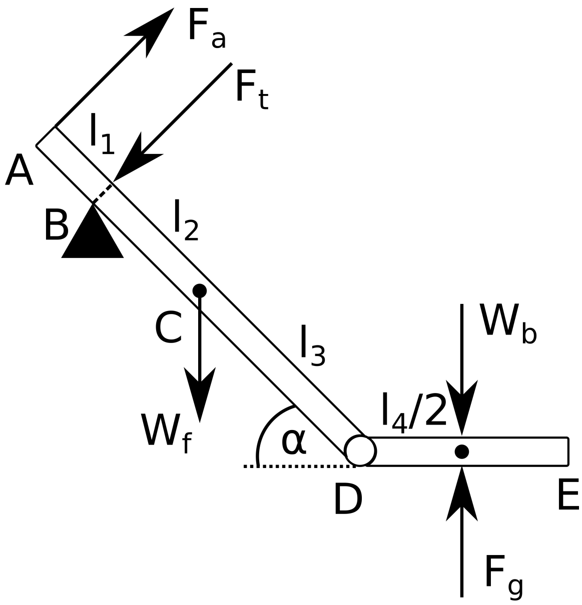


Figure 2.1: Exercise 2.2: Rigid-rod model of a foot for a dynamic move analysis.

where $W_f = m_f g$.

Fig. 2.2 shows the resulting angle dependence of the ratio of the force sustained by the Achilles tendon to the body weight, $r_a = F_a/W_b$, assuming the following inputs:

$$l = (5, 5, 10, 5) \text{ cm}, \tag{2.8}$$
$$m_f = 1 \text{ kg}, \tag{2.9}$$
$$m_b = 80 \text{ kg}. \tag{2.10}$$

The dependence of ratio r_a on the vertical acceleration a is linear, as shown by Eq. 2.7. ■

Dynamic moves are rare in rock-climbing, though, because they consume significant energy, require precise timing and positioning (ideally landing at the apex of the ballistic curve, where vertical velocity vanishes), increase the risk of falling, and can often be replaced by an equivalent sequence of static moves. Static moves typically consist of a precise placement of a foot on a selected foothold, shifting the center of mass as needed to maximize the leverage and efficiency of the subsequent move(s), and standing up on the foot by extending the hip and knee joints. Such motion produces tension primarily in the quads, and secondarily in the calves and hamstrings, while also compressing the cartilage in the patellofemoral joint of the knee, a common cause of chronic and overuse injuries.

Exercise 2.3 Show how the force exerted on the patellofemoral joint, F_P, during a typical static climbing move (extension of knee joint) depends on knee flexion α.

Hints: See Fig. 2.3 by one possible rigid-rod model of the hip-knee-foot system with a symmetry plane at the knee joint (dashed line), bones represented by solid lines (e.g. the hip rod becomes the foot rod after reflection across the symmetry plane), muscles by dotted lines. Assume slow enough movement to invoke conditions of static equilibrium.

Result: Fig. 2.4 shows the resulting angle dependence of the ratio of the force exerted on

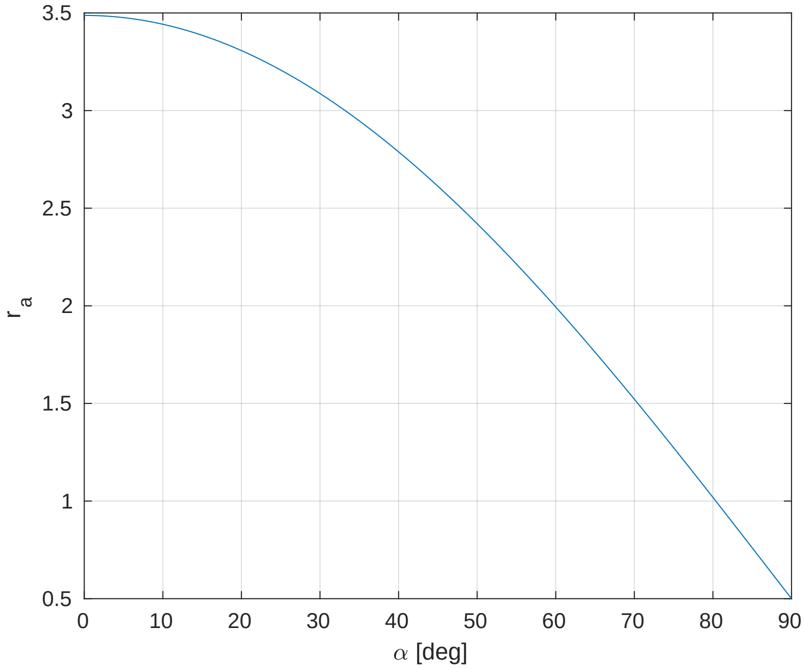


Figure 2.2: Exercise 2.2, dynamic move. The ratio of the force sustained by the Achilles tendon to the body weight, $r_a = F_a/W_b$, vs foot angle α.

the patellofemoral joint to the body weight, $r_p = F_p/W_b$, assuming the following inputs:

$$\sum_{i=1}^{3} l_i = l = 50 \text{ cm}, \tag{2.11}$$

$$\sum_{i=2}^{3} l_i = \frac{l}{3}, \tag{2.12}$$

$$l_3 = 10 \text{ cm}, \tag{2.13}$$

$$a = 10 \text{ cm}, \tag{2.14}$$

$$b = 10 \text{ cm}, \tag{2.15}$$

$$c = 5 \text{ cm}. \tag{2.16}$$

■

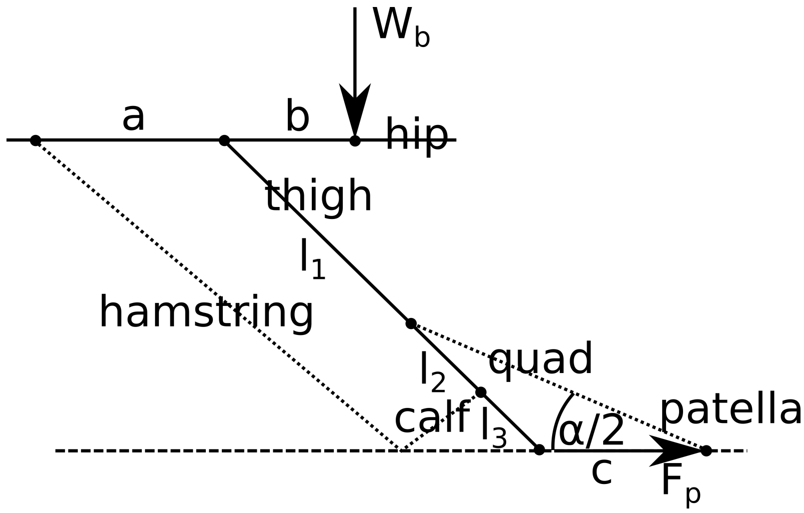


Figure 2.3: Exercise 2.3: Rigid-rod model of a knee for a static move analysis.

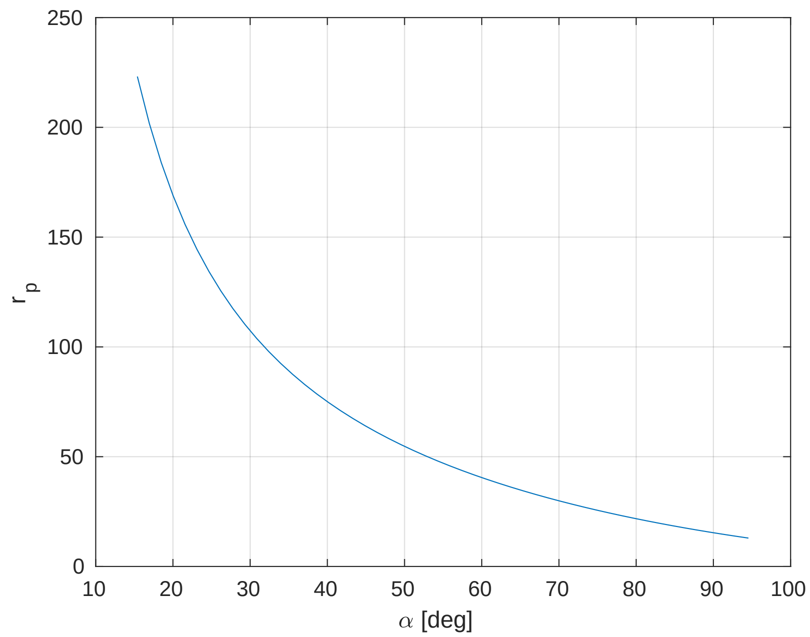
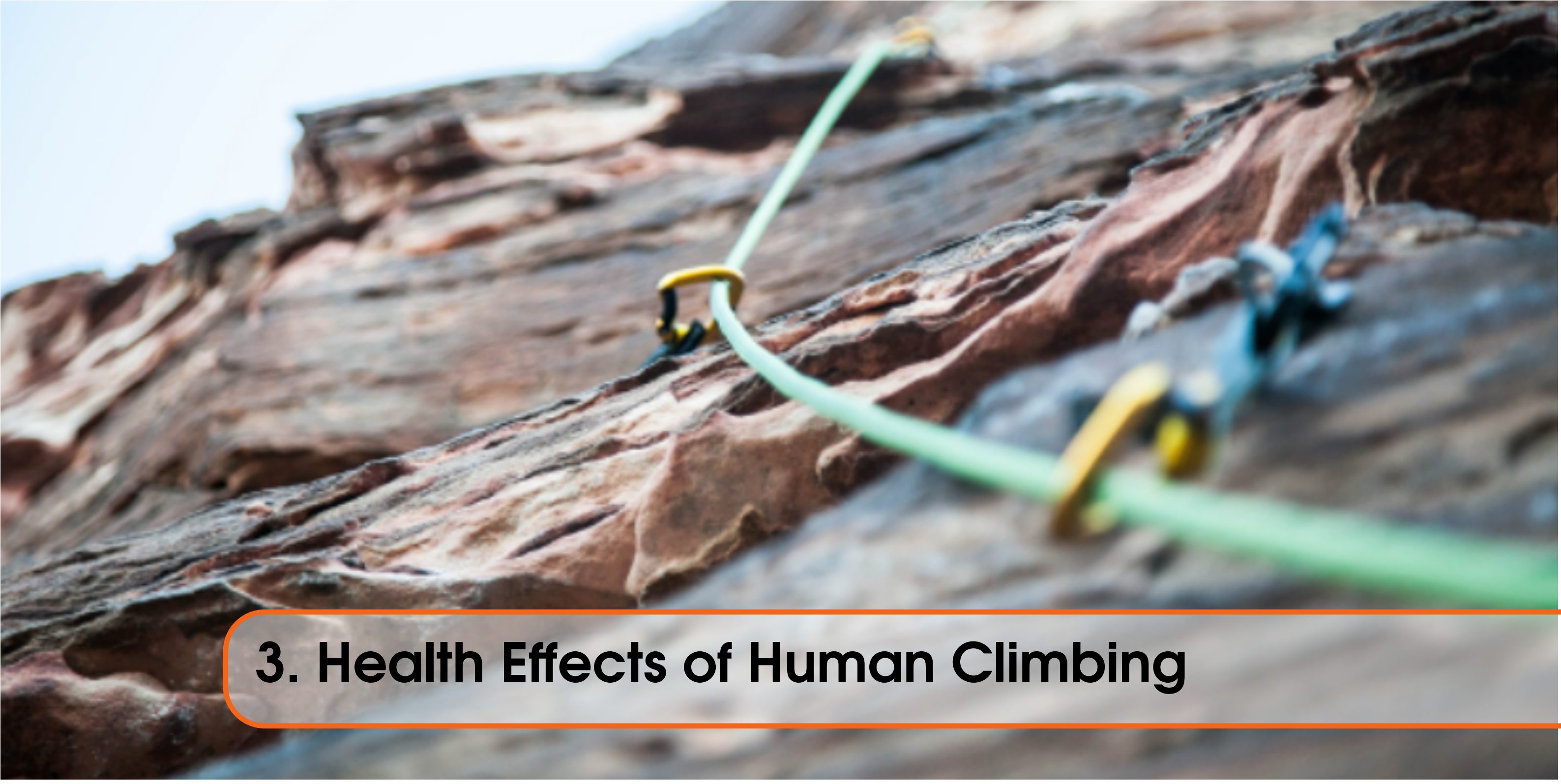


Figure 2.4: Exercise 2.3, static move. The ratio of the force exerted on the patellofemoral joint to the body weight, $r_p = F_P/W_b$, vs knee angle α.



3. Health Effects of Human Climbing

We will discuss physical and medical effects on human health of phenomena most often encountered by rock climbers. These include, for instance, the impact on hard surface after a fall or long periods of being suspended in a seat harness when belaying on multi-pitch routes, but also natural hazards of the mountainous areas, such as high altitudes, extreme temperatures, avalanches, and lightning strikes.

3.1 Finger Injury

Aside from impact traumas, the overuse of soft tissue, especially in fingers, is one of the most common rock-climbing injuries. Often, the annular ligaments (A pulleys) are damaged, such as A2 or A4, as a result of strenuous crimping, which leads to the tearing of the pulley and bowstringing of the flexor tendons. The treatment consists of the standard RICE (rest, ice, compression, elevation) technique for soft tissue injuries, use of NSAIDs and providing sufficient time for the injury to heal.

3.2 Suspension Trauma

Prolonged stay in a harness can induce suspension trauma (related to orthostatic intolerance) due to central ischemic response. This, in turn, can lead to syncope (fainting) and subsequent cerebral hypoxia, which presents the main danger of this cascade of events. The circulatory shock can be prevented by exercising leg muscles while suspended, which activates the venous pumps in the legs. The climbing literature often cautions against the so-called 'reflow syndrome', suggesting that placing the patient into a supine position can overwhelm the right heart due to the increased blood flow, and result in cardiac arrest. There is no medical evidence for such a reflow syndrome (not to be confused with reperfusion injury). The cardiac and/or respiratory arrest, can, however, be induced by the circulatory shock and the associated insufficiency of blood perfusion of tissues, as mentioned above.

3.3 Effects of High Altitude

In human physiology the most prominent effects of high altitude are on respiration. According to the barometric formula the barometric pressure p_b [kPa] decreases exponentially with altitude h [m],

$$p_b = p_b^0 e^{-0.127h}, \tag{3.1}$$

where the sea level pressure $p_b^0 \approx 101$ kPa. This results in the decrease of partial pressures of oxygen (O_2) in the tissues, which are related to the volume fractions by Dalton's law. Under normal circumstances, at sea level, we inspire O_2 at partial pressure $p_{iO_2} \approx 21$ kPa. After being conducted to alveoli ($p_{aO_2} \approx 13.3$ kPa), gas exchange takes places (O_2 and CO_2 diffuse into and from, respectively, pulmonary capillaries). After reaching left heart via pulmonary veins, O_2 circulates via arteries ($p_{AO_2} \approx 12.7$ kPa) from which it diffuses into target tissues. Efficient gas exchange occurs when the ventilation-perfusion ratio is approximately unity, $V_A/Q \approx 1$ (the limiting cases of the ratio vanishing and reaching infinity correspond to, respectively, not ventilated and not perfused alveoli). However, when p_{AO_2} falls below approx. 5 kPa, hypoxia and impairment of cerebral function occurs.

Up to about 7 km of altitude gain the climber's body compensates the O_2 deficit by the so-called O_2 deficiency ventilation: Low p_{aO_2} triggers peripheral chemosensors in glomera aortica and carotica, which stimulate an increase in ventilation via glossopharyngeal (CN IX) and vagus (CN X) nerves. Thus, higher portion of CO_2 is exhaled, driving p_{AO_2} back up according to the alveolar gas equation,

$$p_{AO_2} = p_{iO_2} - \frac{p_{ACO_2}}{RQ}, \tag{3.2}$$

where p_{ACO_2} is the arterial partial pressure of CO_2; RQ, the respiratory quotient, is defined as the ratio of volumetric flow rates of CO_2 to O_2, $RQ = \dot{V}_{CO_2}/\dot{V}_{O_2}$, and depends on person's nutritional state (typical range from 0.7 to 1). The described hyperventilation with excessive exhalation of CO_2 initially leads to respiratory alkalosis, but the climber's body soon compensates for this by increased renal excretion of HCO_3^-, allowing for the O_2 deficiency ventilation to prevail.

Above 7 km of altitude $p_{iO_2} \approx p_b$, necessitating breathing O_2 instead of air. However, high concentration ($p_{iO_2} > 22$ kPa) and/or long duration of O_2 therapy may oxidize the protective surfactant lining the inner alveolar surfaces, prompting alveolar collapse and pulmonary edema, typically manifested by painful breathing, coughing, seizures and unconsciousness.

Above 14 km of altitude pressurized chamber is required for survival, as body fluids near its boiling point (p_b drops below water vapor pressure at body temperature at approx. 20 km of altitude).

3.4 Effects of a Fall

During impact (i.e. the moment when the fall has been stopped by the rope, with vanishing climber's speed), the kinetic energy is deposited into all deformable components of the belay system. While most of the kinetic energy is absorbed by the rope and protection, the rest is dissipated by the climber's body, deforming internal structures due to the imposed stresses. Due to the short time of an impact over which the deformation is allowed to dissipate the materials appear more brittle and thus less resistant to impact (cf. time-temperature superposition in polymer physics for more details). The resulting damage to the climber's body will depend on the dominant modes of deformation of the individual tissues absorbing the impact, and the resulting pathomechanisms.

In an un-suppressed fall on a hard surface the initial impact is typically sustained by lower extremities, resulting in fractures/dislocations of ankles, kness, femurs and/or hips. As the body continues to move, the torso absorbs the impact force, often resulting in rib/sternal fractures and myocardial/pulmonary injuries, including hemorrhage in vital organs. If the energy is deposited further, to the cervical spine and head, then spine injuries, facial fractures and traumatic brain injuries may result. In general, blunt and penetrating forces are the primary mechanisms of injury.

3.5 Effects of Avalanche and Hypothermia

An avalanche can occur when a snowpack (composed of accumulated ground-parallel layers) has a weak layer below a cohesive layer, allowing for relative slippage. The slope must be steep enough to allow snow to accelerate once set in motion, typically at 30-45°angle. The trigger may simply be an increased load (exceeding the snowpack strength) due to recent snowfall, melting of snowpack due to solar radiation, or other meteorological phenomena. The snowpack strength will be influenced by snow properties (size, density, content, morphology) and local air currents and fluxes due to gradients in temperature and water vapor.

The avalanche survival tools include shovel, rescue beacons, probe poles, artificial air pocket devices and airbags. The survival probability is at 90% for those rescued within 15 minutes, then drops to 30% after 30 minutes (acute asphyxiation in the absence of an air pocket), and then again to 7% after 2 hours due to slow asphyxia and hypothermia in a closed air pocket.

Body thermoregulation allows the climber to survive temperature ranges from –50°C to 100°C. Cold induces vasoconstriction and decrease in body's metabolic rate. This can lead to decreased end-organ perfusion, hypovolemia, and, if not corrected, cardiovascular collapse.

Hypothermia further increases changes of circulatory instability (ventricular fibrilation). Treatment includes ABCs (airway, breathing, circulation), cervical-spine precautions, and hypothermia treatment (both external, with removal of wet clothes, shielding from wind, and provision of blankets, and internal, with rewarming with warm IV fluids, gastric/bladder/peritoneal/pleural lavage, and ideally extracorporeal blood warming with cardiopulmonary bypass).

3.6 Effects of Hyperthermia and Dehydration

When wet-bulb temperature of the ambient air exceeds the nominal body temperature of 37°C, the body can no longer cool itself by evaporative cooling (the dominant cooling mechanism in humans). Heat cramps can progress to heat exhaustion and heat stroke (hyperthermia exceeding 41°C, anhidrosis and altered sensorium). While the heat stroke can be fatal, it is preventable. Factors that exacerbate heat exhaustion are those that increase heat production (e.g. infection, stimulants, muscular activity), or decrease heat loss or CNS responses (e.g. drugs, alcohol). The mainstay treatment is rapid reduction of core body temperature. Fluid, electrolyte (e.g. hypernatremia typical in dehydration) and glucose imbalances should be corrected, ideally via IV lines.

3.7 Effects of a Lightning Strike

Lightning, an electric discharge that forms plasma (ionized gas) and a shock wave, is one of the most dangerous natural phenomena encountered by climbers. They occur frequently in the mountains due to high ground elevation, specific atmospheric convection currents and warm and humid conditions. While the mortality is relatively low (10-30%), morbidity is high (70%) and often includes persistent neurologic deficits (vision, hearing, cognitive). The injury can be direct (strike contact) and/or indirect (blunt/concussive trauma due to being thrown). The common cause of mortality is either cardiac arrest (often asystole) or respiratory arrest (due to paralysis of respiratory center). Injuries include superficial burns, fractures, rupture of tympanic membrane and cataracts. Treatment includes Advanced Cardiac Life Support (cardiac arrest), endotracheal intubation (respiratory arrest), fluid therapy (hypotension) and tetanus and antibiotic prophylaxis (burns, open fractures).

If you can hear thunder, you're at risk of getting struck and need to seek a solid shelter (ideally with a functional lightning rod). Staying outdoors is dangerous irrespective of the position your body assumes (squatting, lying down), because after the leader touches the ground the lightning will travel along the ground surface. Avoid tall, isolated objects (increased chance of getting struck). Avoid good conductors (wet ropes, metal gear).

II

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4. Pulley Systems

Commonly employed during rescue operations, including traditionally rock climbing (e.g. hauling an injured follower) or glacier travel (crevasse rescue). Combinations of fixed and movable pulleys are also referred to as block and tackle. While a fixed pulley redirects the force, it offers no mechanical advantage. A mechanical pulley splits the tension into two sections of the rope supporting the pulley, thus offering a mechanical advantage of 2.

Assuming no energy storage and dissipation (mass-less and friction-less pulleys), and no deflection (stretch) in the rope, the mechanical advantage MA of each configuration can be derived from an equilibrium force balance, equating tension forces T in the individual sections of the rope supporting the load, n, with the force of gravity on the load, W (injured climber),

$$nT = W \implies MA \equiv \frac{W}{T} = n. \tag{4.1}$$

Incidentally, denoting the ends of the rope by points A and B, the balance of input and output powers yields

$$MA = \frac{F_b}{F_a} = \frac{v_a}{v_b}, \tag{4.2}$$

where F and v are forces and velocities, respectively.

Connections from the pulleys to the rope are typically fashioned from runners (slings) tied off with a friction (Prusik) knot. Ideally, dedicated low-friction pulleys should be used (friction losses account to approx. 5% up to 30% in ball bearing and bronze bushing pulleys, respectively); carabiners account to approx. 40-60%.

https://www.petzl.com/US/en/Sport/Crevasse-rescue-no–3–haul-systems-for-crevasse-rescue

4.1 2:1, Loop haul

Simple, with minimum gear requirements, and convenient if the accident rope is entrenched, this system is built from a looped rescue rope fitted with a single pulley lowered to the fallen climber, see Fig 4.1. It, however, offers low mechanical advantage, requires the fallen climber to be

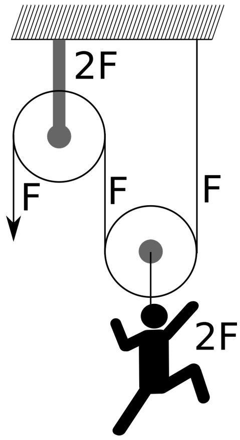


Figure 4.1: Pulley system 2:1

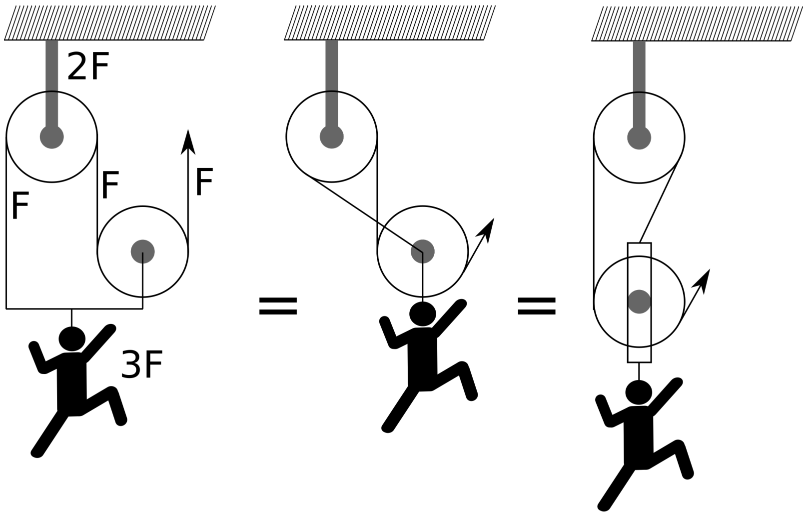


Figure 4.2: Pulley system 3:1

conscious and cooperate, and can easily (with no additional gear) be upgraded into a 3:1 system by interchanging the fixed and moving pulleys.

4.2 3:1, Simple haul

Aka Z-pulley system or gun tackle, popular choice due to its favorable cost/benefit ratio (mechanical advantage vs gear requirements). Owing to a considerable stress placed on the system, the anchors need to be solid. Two pulleys, one fixed and one movable (see Fig. 4.2) reduce the weight of the load by two thirds: one third supported by the rescuer, the other two by the anchor. As the no-free-lunch-theorem implies, the rescuer only raises the load one meter for every every three meters of roped moved through the system.

4.3 5:1 and 7:1 (Double Mariner)

Adding a second movable pulley to the system increased the mechanical advantage to 5 or 7. The mechanical advantage can be increased from 5 (system recommended by local climbing schools) to 7 simply by redirecting the connecting sling from the anchor (fixed pulley) to the rope (Fig. 4.3).

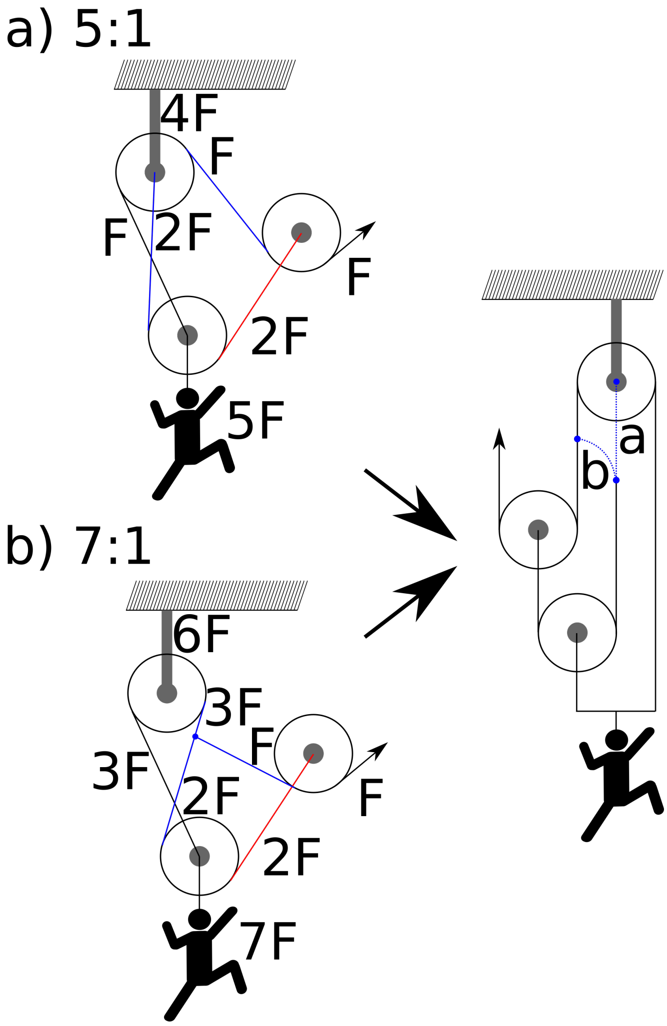


Figure 4.3: Pulley systems 5:1 and 7:1. The blue lines emphasize the differences between the two systems, the red line is typically implemented by a runner.

4.4 Piggybacks

Combining ('piggybacking') the individual pulley systems multiplies the mechanical advantage. E.g. a 2:1 system on a rope coming from a 3:1 system offer a 6:1 theoretical mechanical advantage.

5. Rope Dynamics

A climbing rope has an interior core providing the tensile strength, and a woven exterior sheet to protect the core from abrasion. The dynamic rope elongates appreciably under load (with the maximum elongation specified by the manufacturer), allowing it to absorb the energy of a fall over a longer period of time, reducing the peak impact force and thus the chance of rope failure and bodily harm. During a fall the potential energy gained by the climber ascending is transformed into kinetic energy.

5.1 Rope Model: Impact Force, Fall Factor

In the first approximation, the rope of length l can be modeled as an undamped harmonic oscillator (UHS), with restoring force F proportional to elongation (displacement from equilibrium length) x,

$$F = kx, \tag{5.1}$$

where k is a positive constant (stiffness). Applied to a continuous medium obeying liner elasticity, the Hooke's law above yields

$$\varepsilon E = \sigma, \tag{5.2}$$

stating proportionality between rope's tensile stress σ and its fractional elongation (strain) $\varepsilon = x/l$, with a constant modulus of elasticity E. For a rope of constant cross-sectional area A we define a rope constant $c \equiv EA$, noting that stiffness $k = EA/l$. Using Eq. 5.1 we also define a static displacement x_s of rope under weight $W = mg$ as $x_s = W/k$.

Assuming no energy is lost during impact in the form of heat of plastic deformation, we can write out the mechanical energy conservation for a climber of mass m during the fall as his potential energy is transformed into the tension of the rope,

$$\frac{c}{2l}x_m^2 = mg\left(h + x_m\right), \tag{5.3}$$

where h length of fall before rope elongation occurs and g the gravitational acceleration. Noting that impact force F_c is proportional to maximum rope elongation x_m, $F_c = c\varepsilon_m$, and defining fall factor $f = h/l$, we have

$$\frac{F_c}{W} = 1 + \sqrt{1 + 2f\frac{c}{W}}. \tag{5.4}$$

Thus, the impact force is proportional to the square root of the mass of the climber and its fall factor,

$$F_c \propto \sqrt{fmE}. \tag{5.5}$$

In lead climbing, by definition, the fall factor is limited to $f \leq 2$; in via ferrata, for instance, it can exceed the value of 2 easily.

Unlike other climbing equipment (e.g. carabiners) which is rated by its tensile breaking strength (in kN), the dynamic rope is rated by impact force on its first fall and the number of falls held. While the maximum impact force rating, MIFR, is desirable to be low to prevent depositing excessive forces into climber's body, this is achieved by increasing static elongation, which also increases the probability of a climber impacting the ground. The MIFR is typically around 12 kN, which for an average climber of 80 kg yields the g-force of 15g affecting its body during fall deceleration by the elongating rope. The human tolerance of the g-force depends not only on its magnitude, but also on the length of time it acts as the fall is being arrested, and the direction it acts through the body. E.g. during the arrest of an inverted fall (negative g-force) the increased blood flow to the vessels of the eyes and brain can cause swelling and permanent damage far easier than blood rushing to the feet when subject to a positive g-force.

While an UHS model would suggest as the steady-state solution of the fall problem the periodic oscillations of the climber between its initial height and the height where the rope is under maximum elongation, the motion is in reality dampened due to air resistance and energy absorption and dissipation. Additional shock-absorbers are thus desirable in the belay the system – they increase the elasticity of the system (by lowering the overall elastic modulus E in Eq. 5.5, e.g. harness and anchors) and increase friction between the rope and other components of the belay system. Assuming the standard scenario of n pieces of protection in the belay system, all connected to the rope via carabiners, the kinetic energy is distributed over these components too, adding n additional terms to Eq. 5.3. Approximating the i-th piece of protection and its carabiner by a pulley with friction coefficient μ_i, subject to force F_i, where $i = 1 \ldots n$, we obtain a reduced impact force (cf. Eq. 5.4),

$$\frac{F_c}{W} = 1 + \sqrt{1 + 2f\frac{c}{W} - \sum_{i=1}^{n}\left(\mu_i \frac{F_i}{W}\right)^2}. \tag{5.6}$$

R In a similar vein, an experienced belayer often implements an improvised shock-absorber in the form of a dynamic belay, by allowing for slippage of rope in their belay device during climber's fall, thus trading reduced peak impact force for increased falling distance.

6. Friction

While friction is often neglected in simple engineering calculations, doing so in rock climbing applications often yields results of little theoretical or practical value.

Friction can be beneficial, e.g. allowing a light belayer to hold a heavier climber, or giving tremendous versatility to Prusik knots. Both of these examples can be analyzed through Capstan equation, showing how the holding tension (force) T_1 in a rope wrapped around a cylindrical object (carabiner, another rope) decreases exponentially with the coefficient of friction μ and the number of turns around the object. Denoting contact angle ϕ (one winding is 2π radians) and loading tension T_2, we have

$$T_1 = T_2\, e^{-\mu\phi}. \tag{6.1}$$

In addition to the top-rope pulley friction, the belayer will also be assisted by the friction between his feet and the ground, possibly tilted.

Exercise 6.1 Assume an un-anchored belayer of mass m_1, standing on an inclined plane (angle θ, friction coefficient μ) and connected by a massless rope via a top-rope anchor to the climber of mass m_2, see Fig. 6.1. Suppose we wish to keep the system at rest (acceleration $a = 0$). What is the maximum allowed force F_{max}? Should $F = 0$, what is the maximum allowed mass ratio m_2/m_1? Assume pulleys with no internal friction or inertia. Sketch free-body diagrams if needed.

Solution: Writing out Newton's second law of motion decomposed into x and y components for belayer and climber, respectively, we arrive at

$$T - \mu m_1 g\cos\theta - m_1 g\sin\theta = m_1 a \tag{6.2}$$

$$m_2 g + F - T = m_2 a, \tag{6.3}$$

where T denotes tension in the two rope sections leaving the top pulley (which is loaded with

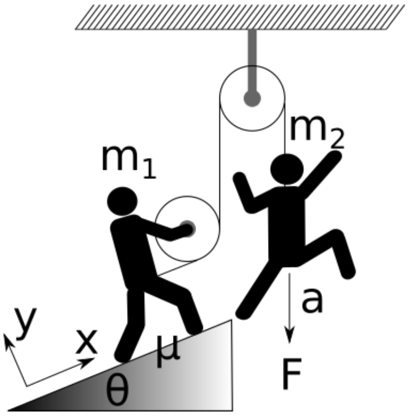


Figure 6.1: Exercise 6.1: Two masses connected by a two-pulley system, with ground friction and inclination included, representing a typical top-roping configuration. Not to scale.

$2T$). Denoting $M = \sum_i m_i$, we obtain

$$a = \frac{(m_2 - m_1 \sin\theta)\, g - m_1 g \mu \cos\theta + F}{M} \tag{6.4}$$

$$T = m_1 \frac{m_2\,(1 + \sin\theta + \mu\cos\theta)\, g + F}{M}. \tag{6.5}$$

The maximum force allowed to keep the system at rest is then

$$F_{max} = (m_1 \sin\theta - m_2)\, g - m_1 g \mu \cos\theta. \tag{6.6}$$

In case of $F = 0$, $a = 0$ is preserved under the condition of

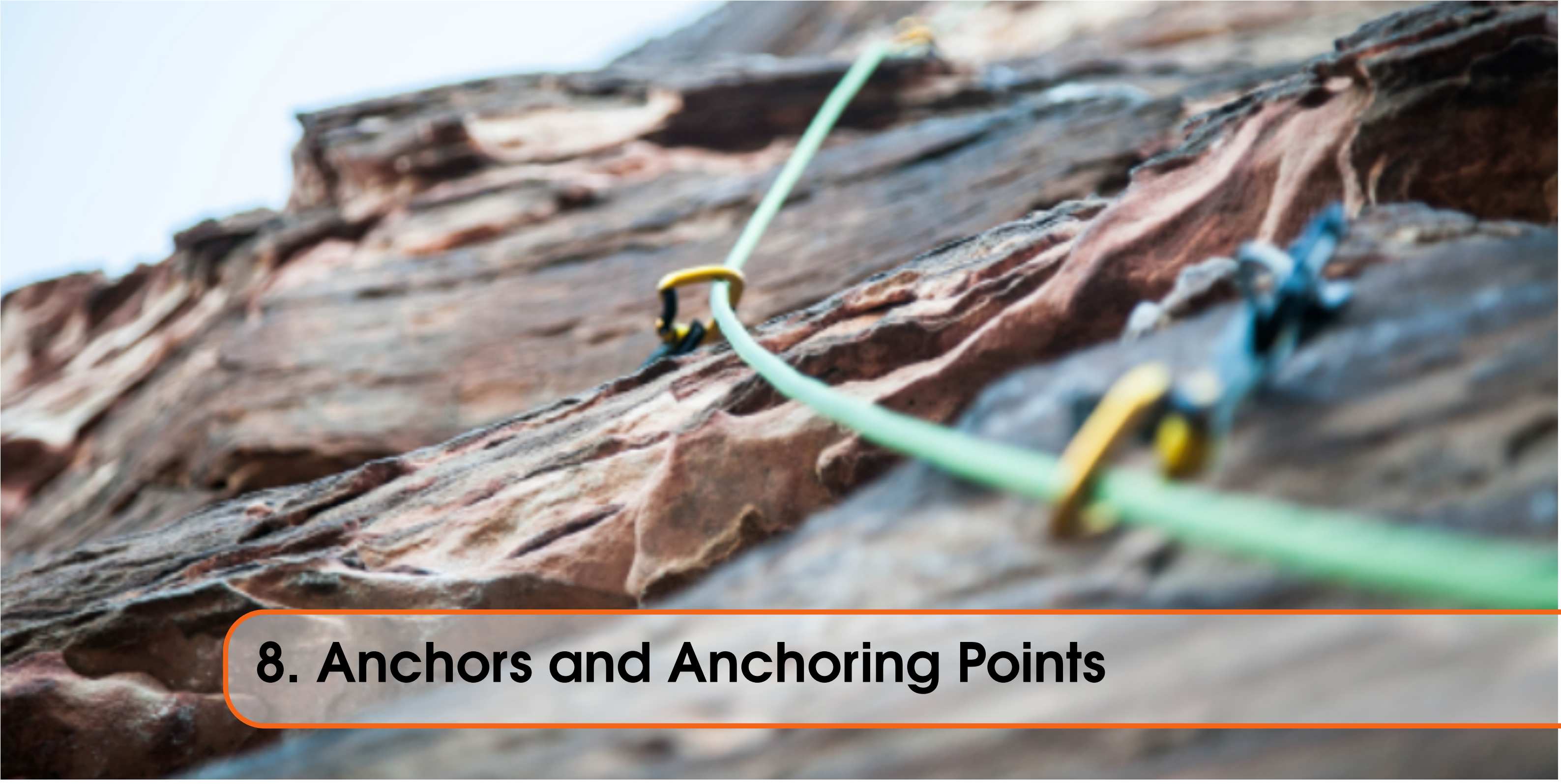
$$\frac{m_2}{m_1} < \sin\theta + \mu\cos\theta. \tag{6.7}$$

We see that for large F and/or m_2/m_1 the belayer is unlikely to arrest a fall even under significant μ and/or θ. A common solution is for the belayer to replace the bottom frictionless pulley with a variable friction pulley (e.g. hip belay, a belay plate, or Munter hitch on a carabiner) to incorporate additional friction into the belay system. More sophisticated devices include internal cams to arrest the fall automatically. Anchoring the belayer is often used as an additional safety measure to ensure the belayer remains continuously in control of his belay device during the climber's fall. ■

Friction, especially excessive, can be detrimental. After leading a pitch it can be difficult for the leader to pull the rope and/or belay the second climber if excessive friction occurs in the belay system, e.g. due too many friction pulleys (quickdraws), rope contacts with the rock (insufficient distance between rope and wall), and/or deviation of rope from straight line (zig-zag protection placement). In addition, significant contact with rock effectively mimics an anchoring point, thereby reducing the available length of the rope l capable to dynamically absorb the fall, thereby increasing the fall factor and impact force.

7. Accessory Cords and Webbing

The tensile breaking strength ratings given for accessory webbing or cords (runners) apply to single strand configurations. As the strength is linearly proportional to the number of strands used, the climber can easily create configurations of the desired strength on the fly. E.g. for a cord with breaking strength of $F_b = x$ kN, a sewn runner (two strands) of the same material will have $F_b = 2x$ kN, and the same runner folded in half (or combined with its duplicate in parallel) will have $F_b = 4x$ kN. While the stitches in the seam can be assumed to have a negligible effect on the strength, a knot, on the other hand, typically reduces the strength by approx. 30-50%, depending on the type of knot and how it was tied. The materials used typically exhibit static characteristics (do not elongate under stress).



8. Anchors and Anchoring Points

Optimal placement of anchoring points (pieces of protection, POPs) is critical for the anchor to perform well. As the anchors are typically subjected to multiple forces, their distribution need to be optimized to prevent anchor failure.

8.1 Distribution of Forces

8.1.1 Anchors

Good climbing anchors adhere to the SERENE principle: solid, efficient, redundant, equalized, no-extension. Assume we have 2-point equalized anchor (created e.g. out of a cordelette, sliding-X, or tied runners), with forces F_1 and F_2 acting on the anchor points, subtended by angle θ. Vector addition yields the resultant force R, whose direction relative to F_1 we denote by β, see Fig. 8.1. Law of cosines dictates

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta, \tag{8.1}$$

which, for a practical approximation of $F = F_1 = F_2$, reduces to

$$\frac{F}{R} = \frac{\sqrt{2}}{2}(1+\cos\theta)^{-\frac{1}{2}}, \tag{8.2}$$

and $\beta = \theta/2$. The plot of Eq. 8.2 shows how the force on each of the two equalized anchors increases from 50% to 100% for $\theta = 0°$ and $\theta = 120°$, respectively, and then continues asymptotically towards positive infinity at $\theta = 180°$. The climbing literature abounds with strong cautions against using anchors with $\theta > \theta_{crit}$, with typical values given as $\theta_{crit} \in (60°, 90°)$. It is important to remember, however, that even anchors with $\theta > \theta_{crit}$ can be used in cases where anchoring points are deemed strong enough to bear the additional load, because such an equalization still provides redundancy.

8.1.2 Belayer

We can extend the analysis above to study the force distribution during climber's fall not only on anchors, but also on the belayer. Assume that in Fig. 8.1 the forces F_1 and F_2 denote the

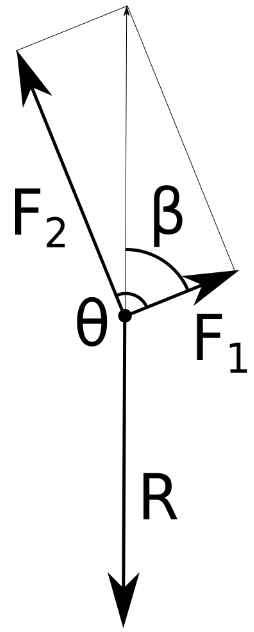


Figure 8.1: 2-point equalized anchor.

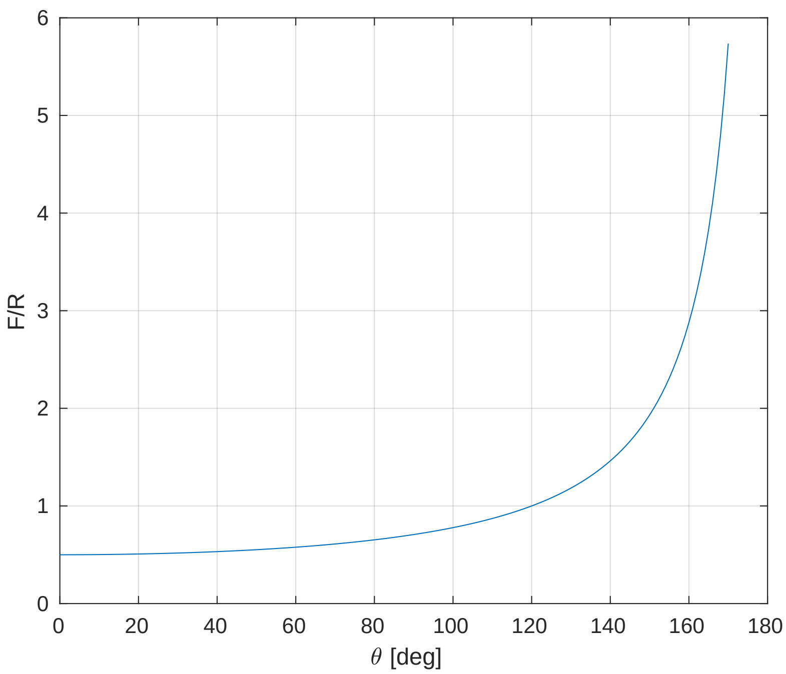


Figure 8.2: Fraction of resultant force on each of two equalized anchors, F/R, as a function of θ.

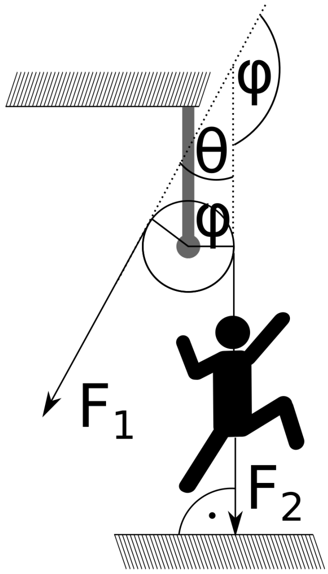


Figure 8.3: The Capstan equation 6.1 shows how the load ratio k depends on the coefficient of friction between the rope and the carabiner, μ, and on the contact angle $\phi = \pi - \theta$.

holding tension by the belayer and the loading tension by the fallen climber, respectively, with the intersection point representing a top-most anchor (e.g. a top-rope anchor, or the last piece of protection placed by the leader). Denoting the fraction of the climber's load supported by the belayer as $k = F_1/F_2$, and the fraction of the climber's load supported by the top-most anchor as R/F_2, Eq. 8.1 yields

$$\frac{R}{F_2} = \sqrt{k^2 + 2k\cos\theta + 1}. \tag{8.3}$$

The Capstan equation 6.1 further suggests the dependence of k on the coefficient of friction between the rope and the carabiner, μ, and on the contact angle $\phi = \pi - \theta$, see Fig. 8.3. Specifically, we have

$$k = e^{-\mu\phi}. \tag{8.4}$$

By combining Eq. 8.3 and Eq. 8.4 we can show that as the angle θ at the top-most anchor increases, then:

1. The belayer absorbs increasingly larger force k due to the loss of contact area (and thus friction) between rope and carabiner (decreasing ϕ). The only exception is a friction-less pulley ($\mu = 0$), where the belayer has to support the full loading tension exerted by the climber, irrespective of the angle ϕ.
2. The resultant force on the anchor, R/F_2, decreases, with the curve gradient proportional to the friction coefficient μ; a typical carabiner-rope system has $\mu \approx 0.4$.

8.2 Anchoring Points

An anchor depends on a solid placement of anchoring points, whicch should be guided by

1. Rock quality
   Classification of rock (igneous/sedimentary/metamorphic) aids in evaluation of its basic physio-chemical properties. Use both visual and tactile inspection to estimate rock's physical

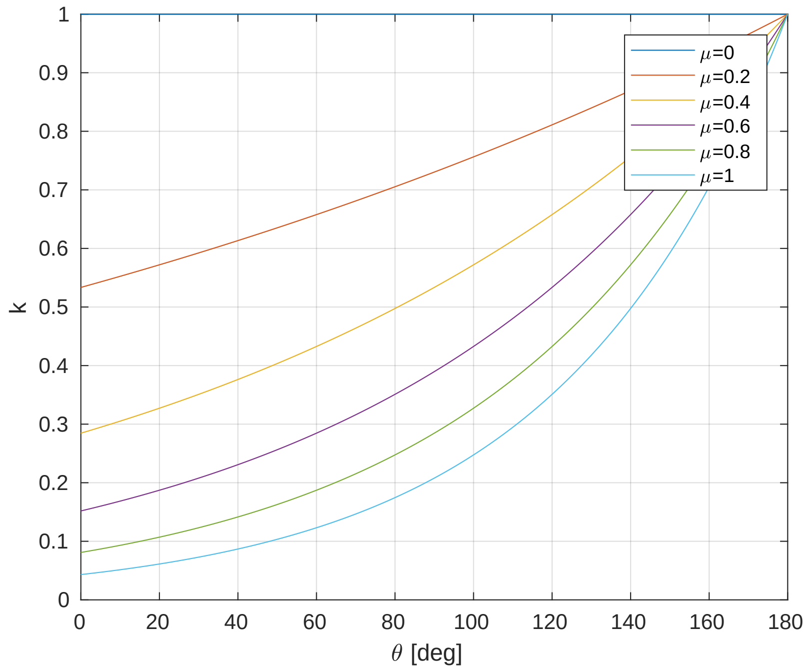


Figure 8.4: Force on the belayer (relative to loading tension), k, as a function of θ.

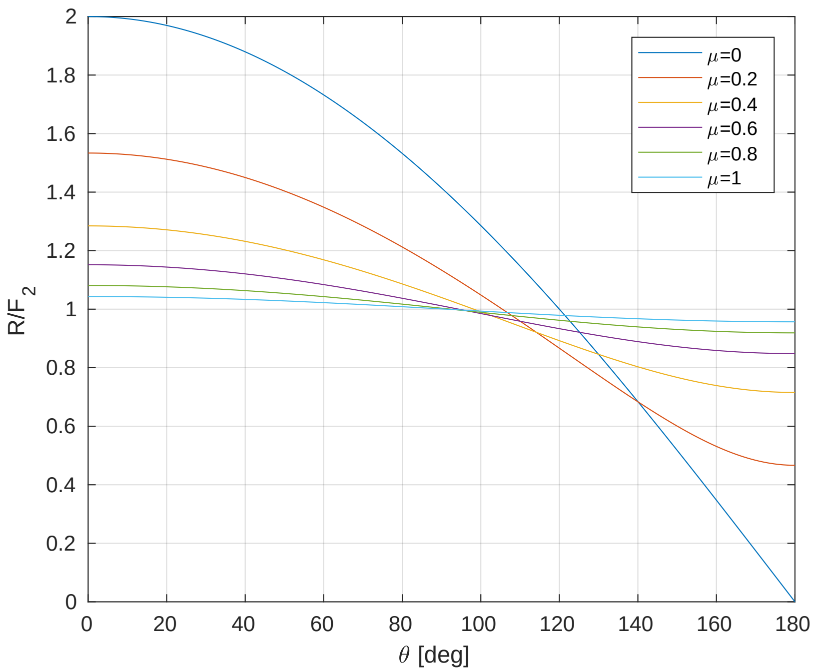


Figure 8.5: Resultant force on the top-most anchor (relative to loading tension), R/F_2, as a function of θ.

characteristics, first in isolation (hardness, fracture), and then taking into account its environment (external stresses, effects of local erosion etc.). E.g. a firm, solid and intact rock is preferable to loose rock affected by erosion.

While rock and POPs are typically strong (able to withstand significant loads without failure), they are also brittle, i.e. when they do fail, they break (off) rather than undergo plastic deformation. In addition, the resistance to stresses is typically anisotropic, i.e. dependent on the direction in which the force is acting. E.g. sedimentary rocks, such as limestone, can be stronger in compression and weaker in tension, due to the structure of the material (particle deposits cemented together). This makes it possible for a POP (e.g. a chock) under significant load to shear off a piece of rock and come loose.

2. Rock-POP interaction

   The rock and the placed POPs create a coupled multi-body system, typically subjected to multiple contacts and friction. Estimate the dynamic interactions of this system under multi-axial rope movement and climber's fall by asking yourself questions such as
   - What will be the magnitude and direction of forces and stresses (both tensile and shear) acting on the coupled system and inducing the motion and deformation of POPs relative to the rock?
   - What are the variables that influence these stresses (e.g. the rock-POP interfacial area, the shapes of mating surfaces etc.)?
   - Will the induced motion of the POP be arrested by sufficient friction and/or steric effects (volume exclusion) of the rock, or does the system need to be expanded by additional POPs to counteract the induced motion?

Exercise 8.1 Assume a typical fall of on a standard dynamic rope. The leader of weight m placed all the pieces of protection in a straight line, except the last one, which deviates by angle θ. What is the highest load in the system, and how does it depend on m and θ?

Solution: We will represent a typical fall by $f = 1$ and the standard dynamic rope by diameter $d = 10$ mm and elastic modulus $E = 30$ kN. Referring to Fig. 8.6 we represent the series of straight-line pieces by a single pulley with contact angle θ_1 and resultant force R_1. We assume all the quickdraw (carabiner) protection contacts to be equivalent and represented by a pulleys with friction coefficients $\mu \equiv \mu_1 = \mu_2 = 0.5$.

The highest load in the system is obviously acting on the top-most anchor with contact angle θ_2 and resultant force R_2, with $R_2 > R_1$. The belayer and climber are subject to impact forces F_1 and F_3, respectively, with F_2 acting in between the two pulleys. Since F_1 and F_3 act along parallel lines, the consecutive angles along the transversal F_2 are complementary, i.e. $\theta_1 + \theta_2 = \pi$. Our goal is to find dependence of R_2 on m and θ_2.

Applying the law of cosines, Eq. 8.3, and the Capstan equation, Eq. 8.4, to both pulleys, we obtain the four equations

$$\frac{R_2}{F_3} = \sqrt{k_2^2 + 2k_2 \cos\theta_2 + 1}, \quad k_2 = e^{-\mu\phi_2} = \frac{F_2}{F_3}, \tag{8.5}$$

$$\frac{R_1}{F_2} = \sqrt{k_1^2 + 2k_1 \cos\theta_1 + 1}, \quad k_1 = e^{-\mu\phi_1} = \frac{F_1}{F_2}, \tag{8.6}$$

where $\phi_2 = \pi - \theta_2$ and $\phi_1 = \pi - \theta_1 = \theta_2$. Below we simplify the notation using $\theta \equiv \theta_2$. The fifth equation is given by the impact force on the climber, Eq. 5.6,

$$\frac{F_3}{W} = 1 + \sqrt{1 + 2f\frac{c}{W} - \sum_{i=1}^{2}\left(\mu_i \frac{R_i}{W}\right)^2}, \tag{8.7}$$

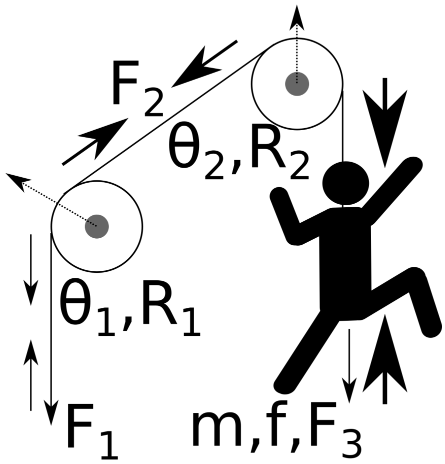


Figure 8.6: Exercise 8.1 setup. Analysis of critical load R_2, and its dependence on climber's weight m and contact angle θ_2, in a typical lead climbing situation.

which, upon substitutions, yields an implicit function for F_2, $g(F_2) = 0$. Upon simplification, we obtain a quadratic equation

$$AF_2^2 + BF_2 + C = 0, \tag{8.8}$$

with

$$AW^2 = \pm\mu^2 s(\mu,\theta) + e^{-\mu(\pi-\theta)} \tag{8.9}$$
$$BW = -2e^{\mu(\pi-\theta)} \tag{8.10}$$
$$C = \mp\left(2f\frac{c}{W} + 1\right) - 1, \tag{8.11}$$

where $W = mg$, $c = E\pi(d/2)^2$, and

$$s(\mu,\theta) = e^{2\mu(\pi-\theta)} + e^{-2\mu\theta} + 2\cos\theta\left[e^{\mu(\pi-\theta)} - e^{-\mu\theta}\right] + 2. \tag{8.12}$$

Upon eliminating unphysical roots, Eq. 8.8 yields dependence on m and θ_2, as shown in Fig. 8.7 and Fig. 8.8, respectively.

You can perform a sensitivity analysis to show the results are little affected by fall factor f, friction coefficient μ or rope constant c. ■

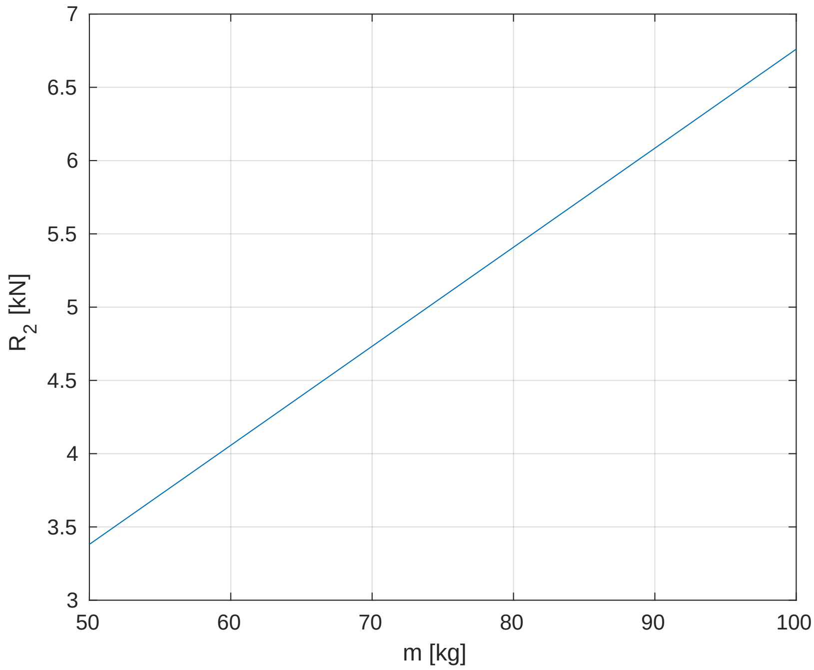


Figure 8.7: The top anchor is subjected to load R_2 with linear dependence on climber's weight m.

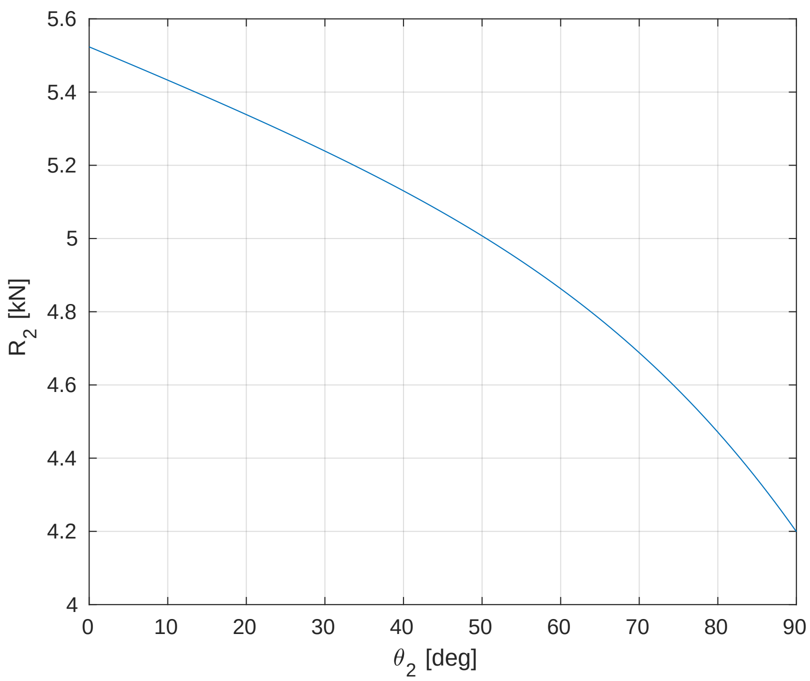


Figure 8.8: The top anchor is subjected to load R_2, which is inversely proportional to contact angle θ_2.

9. Metal Fatigue and Fracture

Critical climbing gear, such as carabiners and belay plates, are subjected to repeated cyclic loadings, where the internal stress fluctuates, in time, around a nominal value with a given frequency. Made of ductile metals, such as steel or aluminum, they may undergo progressive damage in the form of crack propagation, i.e. fatigue. Failure can the occur at load lower than yield strength of the material under static load.

Micro-fractures are typically omnipresent and not necessarily a case for concern (e.g. an average wing of a commercial aircraft, certificated to highest safety standards, typically exhibits billions of micro-fractures while still presenting an extremely low risk of material failure). They arise due to manufacturing imperfections, or due to persistent dislocation and slip movements at microscopic levels. However, these discontinuities often serve as locations of increased stresses and thus initial points of crack formation. Once a crack forms, it typically grows along planes of high shear stress until a critical reduction in load-bearing area occurs, leading to failure (fracture).

The fatigue is typically described by so-called S-N curves, where S and N signify, respectively, stress (including normal, such as compressive or tensile, or tangential, such as shear) and time to failure. These are typically governed by power laws, such as

$$N\sigma^{m_1} = const, \tag{9.1}$$

where σ is the stress amplitude, N the number of cycles to failure and m_1 a material specific empirical constant. Thus plotting the stress against $\log(N)$ yields a monotonically decreasing linear dependence, as is the case for aluminum. For steel, however, the decrease gradually flattens out, asymptotically approaching a 'fatigue limit', which means theoretically infinite life is possible if stresses are kept under this limit.

Once a crack of length a forms (can be observed by e.g. X-ray photography), its growth rate will also be governed by a double power law, typically in the form of

$$\frac{da}{dN} \propto \sigma^{m_2} a^{m_3}, \tag{9.2}$$

where m's are empirical constants, with $m_2 \in (2,4)$, $m_3 \in (1,2)$. The presence and size of grains and inclusions will also have and effect, as well as some degree of randomness. For more detailed analyses consult texts on fracture mechanics.

The strength rating of a carabiner for axial loads is typically a third of its nominal value when the gate is open (e.g. 24 kN and 7 kN for closed and open configuration). This is because a closed frame is much stronger and will require a large axial load to be stretched along the major axis. In the open configuration, however, the resulting shape can bend (and rupture), typically in the flexure points, under much smaller loads. The S-N curves discussed above apply equally to carabiners in both configurations.

For low stresses some microscopic plasticity occurs. For long-term stresses at high temperatures (such as during rappeling) can exhibit a permanent deformation called creep. The steady-state rate of deformation, strain/creep rate $d\varepsilon/dt$, is a function of the material's properties, and is proportional to the applied stress σ and Boltzmann factor,

$$\frac{d\varepsilon}{dt} \propto \sigma^{m_4} e^{-\frac{Q}{k_b T}}, \tag{9.3}$$

where m_4 is an empirical constant, Q creep activation energy, k_b Boltzmann constant and T temperature.

How should you deal with the potential of fatigue in your gear?

- Theoretically, you can use the concepts of Physics of failure and leverage your knowledge of the mechanisms to predict the reliability and/or failure of your gear. One possibility, common in aerospace industry, is to assume a probability distribution of crack sizes and model the crack growth rates, ensuring the probability of failure remains below an acceptable level.
- Practically, you should always build your belay systems in a fail-safe and fault-tolerant manner, adding redundancy, and preventing a single point of failure.

10. Whiplash Effect

When a carabiner suddenly decelerates, the resulting inertia causes the gate to open momentarily. With the gate open,

- the carabiner body strength decreases (typically to a third of the closed-gate rating), and
- the probability of the rope unclipping increases.

Such a deceleration can occur when a carabiner (e.g. in a quickdraw) impacts the rock and/or due to a sudden downward tension of the rope threaded through the carabiner, as the rope becomes loaded by the weight of the fallen climber. From the Newton's equation for uniform acceleration,

$$F = 0.5mv^2/s, \tag{10.1}$$

we see that the gate inertia is proportional to its mass m and the square of its initial velocity v, and inversely proportional to the stopping distance s. While the velocity and distance will depend on the dynamics and configuration of the fall, the mass can be lowered by using a wiregate carabiner. The situation is analogous to the whiplash injury arising from the relative motion of the head and torso in rear-end automobile collisions.

Another potential cause of a carabiner gate opening are the vibrations of the rope (the so-called gate flutter), yet such situations are extremely rare in rock climbing and thus mostly of academic interest only. The phenomenon is similar to the aeroelastic flutter (responsible for the infamous Tacoma Narrows Bridge collapse), although here the positive feedback arises between the gate deflection and the force transmitted by a vibrating string (rope), rather than the fluid flow.

11. SLCD analysis

Here we present a simple mathematical model, based on the work of Dave Custer from MIT, for the most popular type of rock-climbing anchor, i.e. the spring-loaded camming device (SLCD). The working mechanism is simple: the downward pulling force wedges the cams into the rock (acting normal to the rock surface and accounting for the friction-less contact) and subsequent friction between the SLCD material and the rock, acting tangential to the rock surface, prevents the SLCD from slipping.

11.1 Frictional contact

A single cam wedged in a parallel crack can be approximated by a rigid rod, with one end (A) free to rotate (cam's axle) and the other end (B) in shearing contact with a rigid wall, see Fig. 11.1 The horizontal forces due to opposing actions of paired cams cancel out at point A, leaving only the vertical component in the form of the applied force, F_a. At point B, both normal and tangential (frictional) force components prevail, limiting the traction at the rock surface. The stress boundary conditions at the cam-rock interface reduce to the conditions of static equilibrium for a closed system, i.e. vanishing resultant (total) force and torque, yielding the ratio between the force components

$$\frac{F_a}{F_n} = \mu_c, \tag{11.1}$$

and the required friction coefficient μ,

$$\mu \geq \mu_c, \tag{11.2}$$

where we define the critical friction coefficient $\mu_c \equiv \tan\beta$. The camming angle β is typically around 14°, yielding $F_a/F_n \sim 1/4$. Thus, the cam's mechanical advantage of 4-to-1 translates each unit of vertical loading by the fallen climber into 4 units of horizontal force compressing the cam against the rock surface. A typical friction coefficient at the aluminum-granite interface is $\mu = 0.38$, limiting the maximum camming angle β to about 20°.

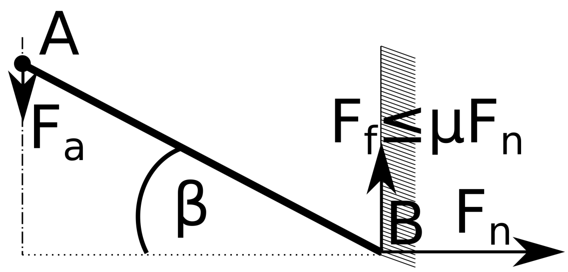


Figure 11.1: Rigid rod model of a single cam mounted on a common axle at point A where the loading force is applied. At the cam-rock interface (point B) the cam shears against the rock with the frictional (tangential) force constrained to prevent the cam from slipping.

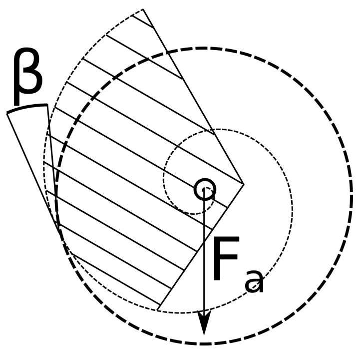


Figure 11.2: Cam shape approximates the logarithmic spiral with the polar slope angle (camming angle) $\beta = const$.

Suppose we now wish to translate this simplified rigid-rod abstraction into a 2D shape. We can parametrize the rotation of the rod around the axle subject to the constraint of keeping the polar slope angle β constant. Because the polar slope angle is defined as the angle between the tangents to the desired curve (ideally conforming to the rock surface) and the circle of the same radius, such constraint maximizes the chances of conforming to the irregular rock surface. To meet this constraint the length of the rod must grow exponentially, with the end point B tracing a logarithmic spiral, written in polar coordinates (r, ϕ) as

$$r \propto ae^{\mu_c \phi}. \tag{11.3}$$

The logarithmic spiral is thus the general shape which most SLCDs approximate, as shown in Fig. 11.2.

11.2 Friction-less contact

After deriving the crucial frictional condition represented by Eq. 11.2, we can extend the model further by asking how the maximum applied force that the SLCD can sustain is influenced by the properties (material and geometrical) of the cam and the rock. The answer lies in focusing on friction-less (normal) forces, allowing for the deformation of the rock as the SLCD bites into its surface and creates a slight indentation.

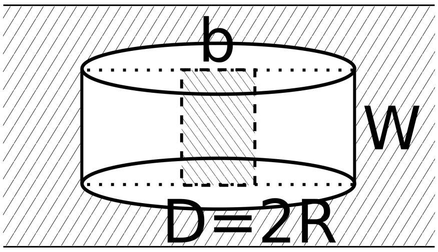


Figure 11.3: Cam approximated by cylindrical surface pressing against the planar rock surface, with contact area $A = bW$.

We can locally approximate the curvature of the spiral by that of the circle; then, the cam can be represented by a cylindrical surface of radius R and diameter $D = 2R$ pressing, in the normal direction, against a rock-surface plane, see Fig. 11.3 As the surfaces come into contact across the area A, they both deform slightly under the imposed load, creating an indentation into the rock surface of depth d.

As a starting point of the analysis we can use the equation for Hertz contact stress in case of two cylinders with parallel axes (or, equivalently, equations for the bearing pressure, i.e. male cylinder – female cylinder contact), which gives the width of the contact surface b as proportional to the square root of the indentation depth d,

$$b = \sqrt{Rd}, \tag{11.4}$$

and the indentation depth d as proportional to the normal force F_n,

$$d \sim \frac{4F_n}{\pi W E^*}, \tag{11.5}$$

where W and E^* denote, respectively, the width of the cam (length of the approximating cylinder) and the contact modulus of elasticity. E^* depends on the material properties of both the cam and the rock, namely their elastic moduli E and Poisson's ratios ν (ratio of transverse expansion to axial compression), specifically

$$\frac{1}{E^*} = \sum_{i=1}^{2} \frac{1-\nu_i^2}{E_i}, \tag{11.6}$$

where the subscripts $i = 1$ and 2 denote, respectively, cam and rock materials. Thus, we have

$$b = \sqrt{\frac{2F_n D}{\pi W E^*}}. \tag{11.7}$$

More accurately, D in Eq. 11.7 should represent the relative radius of curvature D^*, defined as

$$\frac{1}{D^*} = \sum_{i=1}^{2} \frac{1}{D_i}. \tag{11.8}$$

For a planar rock surface with infinite radius of curvature, though, we obtain the identity $D^* = D_1 \equiv D$.

SLCD consists of n cams (typically $n = 4$); the contact area A at the interface between a single cam and the rock surface follows from Eq. 11.7, yielding

$$\frac{A}{n} \equiv bW = \sqrt{\frac{4F_n RW}{\pi E^*}}. \tag{11.9}$$

Defining the maximum shear strength as

$$\tau_{max} = \frac{F_{a,max}}{A}, \tag{11.10}$$

where $F_{a,max}$ is the maximum applied force that can be sustained by the SLCD, we can combine the equations of frictional mechanics (Eq. 11.1) and friction-less mechanics (Eqs. 11.9,11.10) to obtain

$$F_{a,max} = \frac{4\tau_{max}^2}{\pi E^*} \times \frac{RWn^2}{\mu_c}, \tag{11.11}$$

where the first and second fractions on the right-hand-side account for, respectively, the material properties and cam design (geometry).

The validity of Hertzian contact mechanics outlined above is based on several assumptions, most importantly that the contacting bodies have dissimilar shapes (non-conforming contact) and characteristic dimension much larger than the contact area (small strains); these are both reasonable assumptions in our case.

Exercise 11.1 Show how the maximum loading force that can be sustained by a typical SLCD depends on the cam radius and type of rock, accounting for both tangential friction and the deformation of the cams and the rock due to the normal wedging force.

Solution: A typical SLCD can be assumed to consist of $n = 4$ cams, with the can lobe width approx. $W = 1$ cm and camming angle $\beta = 14°$. The cams are made of aluminum alloy with shear strength approx. $\tau_{max} = 100$ MPa (order of magnitude estimate), elastic modulus $E = 70$ GPa and Poisson's ratio $\nu = 0.33$. The majority of rock-climbing crags feature granite, limestone and sandstone formations, with the average values $E = (50, 30, 10)$ and $\nu = (0.2, 0.26, 0.3)$, respectively.

Using Eqs. 11.11 and 11.6 we obtain the dependence depicted in Fig. 11.4. We see that going from granite to limestone and sandstone, the elastic modulus E decreases, which in turn increases the indentation depth d and thus the contact area A. The linear dependence of the maximum loading force on the cam radius confirms our intuition.

Because of the numerous simplifications inherent in the model, and because most of the input data are order of magnitude estimates, the results should only be interpreted qualitatively. Furthermore, the friction coefficient is also influenced by humidity of the surfaces and the heterogeneous nature of the rock. ■

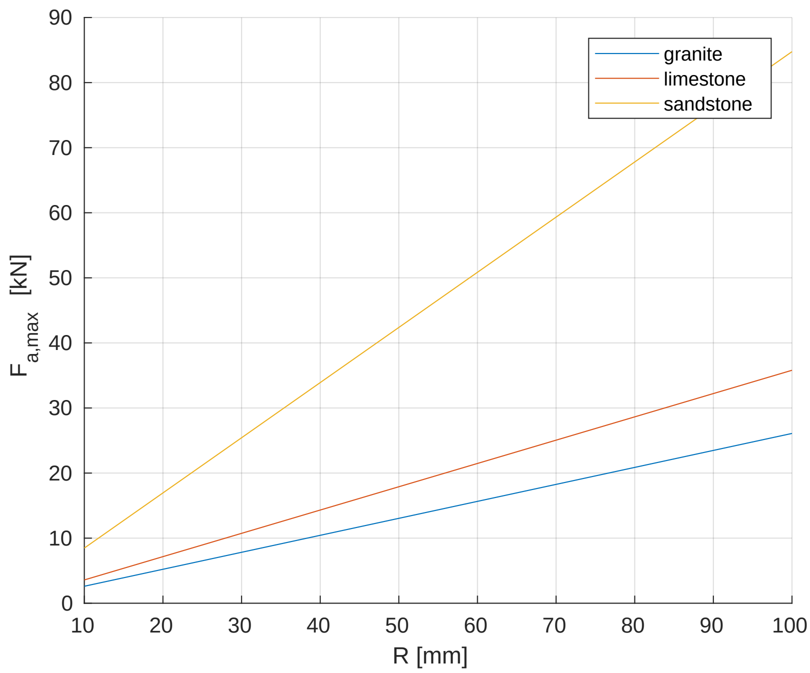


Figure 11.4: Maximum loading force on an typical 4-cam aluminum SLCD versus cam lobe radius *R*. On granite, limestone and sandstone.

The Environment

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12.1 Storms

12.2 Wet Rubber

12. Meteorology

12.1 Storms

A large obstacle such as a mountain separates the space into multiple regions. These regions will receive different solar irradiance (e.g. with one mountain face in the shadow and the opposite one in the sun), creating spatial inhomogeneity in the local tropospheric gas. When air masses with different properties (temperature, pressure, humidity) are in such close proximity to one another, a small disturbance suffices for their collision, which is why the weather will tend to be unstable and unpredictable. Further, the shape of the mountain will guide the currents vertically rather than horizontally. Warm, moist air will ascend along the slopes of the mountain, cooling down in the process to the dew point, creating water condensation and, at sufficient height, ice crystals (glaciation). These precipitation particles will have nonuniform sizes and weights, which alters their convective trajectories. This leads to their collisions, with subsequent charge acquisition and separation (cloud electrification).

A number of such spatial obstacles (mountainous range) will amplify the above-described effects.

The stability and moisture content of the air masses surrounding a mountaineer will vary throughout the day. Carefully observing these changes can help the person make a short-term weather forecast sufficiently accurate to make crucial decisions about the planned climb. Cumulus clouds sprouting vigorously upwards can be one sign of thunderstorm formation. The higher the moisture, the lower they appear. The higher the instability, the faster the upward growth.

12.2 Wet Rubber

Unlike hiking shoes, the climbing shoes have no tread, so as to maximize the contact area available at footholds. A water layer between the rock and the climbing shoe (present e.g. during or after precipitation) will reduce the friction and significantly reduce the grip.

IV Advanced Techniques

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In this part we turn from applied physics to some examples of advanced climbing methodology, where the climber particularly benefits from the knowledge of the underlying physical mechanisms discussed in the preceding parts.

13. Rope-Based Belays

Building belays from the rope is efficient and thus popular among seasoned climbers. There are several advantages:

- Fast to set up.
- Safe (dynamic rope vs static sling).
- Flexible (distance can be fine-tuned).
- Economical (no extra material, such as cords or slings, is required, only the rope and ideally carabiners).

The disadvantage: rope-based belay may complicate self-rescue.

13.1 Belay Near Anchor

The rope is clove-hitched to the POPs (pieces of protection), and the resulting bights of rope are tied together into a central point (e.g. overhand/figure 8/figure 9 on a bight). If you are not swinging leads, the second simply ties into carabiners-on-carabiners, so that the leader can remove the clove-hitched belay before starting on the next pitch.

13.2 Belay Away From Anchor

If you wish/need to keep distance between your belay stance and the top anchor, you have several options of attaching the rope to the anchor:

1. Loop through. Fine-tuning the climber-anchor distance is easy, but may be unsafe. Tie off at the harness (with or without a carabiner).
2. Clove hitch. The climber-anchor distance is set and cannot be tuned (once anchor is out of reach), but is safe (climber is tethered).
3. Munter hitch. The climber-anchor distance can be tuned, and climber is relatively safe (on Munter self-belay).

In all three cases the central point is again built with the rope (e.g. overhand/figure 8/figure 9 on a bight), and the second is belayed from it using the UIAA belay method (Munter hitch on and HMS carabiner). The central point can be created on either of the two available strands of the rope –

the choice depends on personal preference and the given belay configuration (standing or hanging belay, relative height between anchor and leader's harness), and should be based on the prediction of rope movement during the fall.

13.3 Word Of Caution

If you don't use rope-based belays, avoid using the popular Sliding-X without the limiting knots (e.g. overhands). While scoring high on equalisation, the extension in a fall generates impact forces high enough to rupture the slings and/or disengage the POPs.

14. Rope Soloing

Similar to regular climbing, solo roped climbing can be divided into top-roping (limited) and lead climbing (general).

14.1 Top-Roping

Solo top-roping is a fairly safe and common practice, with multiple variations. All the techniques used require:

1. Setting up a top anchor and fixing the rope to it. Any proven anchoring knot is sufficient, such as figure eight knot (rewoven/on a bight), bunny ears, clove hitch). The top anchor should be SERENE 8.1.1. Climbing on either one (fast and efficient) or two (redundant) rope strands is common. The second rope typically serves as a back-up (with backup knots for the climber to clip in, or with a separate self-belay device attached).
2. Setting up a self-belay. Similar to multiple ropes, using multiple devices (including self-belays and carabiners) and/or methods of their attachment to the rope and climber's body increases redundancy while reducing climbing efficiency (no free lunch). Commonly used devices include Petzl MicroTraxion and/or Shunt.

Being able to first reach the top without leading is the primary limitation of this technique, which makes it suitable mostly to terrains with easily accessible top anchors. Multi-pitch and alpine routes invariably require the more general and versatile technique of solo leading.

14.2 Leading

Leading solo is the ultimate roped discipline. Each pitch must be effectively climbed three times:

1. Set up a bottom anchor and lead up.
2. Set up a top anchor, fix the rope and rappel down.
3. Disassemble the bottom anchor and prusik/jumar back up.

Rinse and repeat for each pitch. Currently, efficient self-belay devices for solo leading are in short supply (Rock Exotica's Soloist and Silent Partner are no longer in production), the most common solution is thus the traditional Yosemite's approach of tying up with a clove hitch, which is both safe and awkward. Safety of the climb can be increased in several ways, e.g. by:

- Using two steel carabiners threaded through multiple harness points to tie in the main (sliding) clove hitch.
- Periodically retying a backup clove hitch (clip into a new one before removing the old one).
- Tying in to the end of the rope.

From the above it follows that solo leading requires both physical endurance and mental stamina. Neat and efficient rope management is crucial to a steady progress.

The primary limitation of this technique is its reliance on a solid bottom anchor, which may be difficult to set up at certain pitches.

15. Horizontal Traverse

Lateral movement on the rock, either while climbing up or abseiling, is often necessary in alpine conditions. E.g. a section of the wall too difficult or dangerous to climb may need to be traversed prior to resuming the vertical climb. Also the abseil often deviates from the vertical, either due to easier terrain, or because the next abseil station is to the side.

Two common techniques include pendulums and tension traverses. The pendulums in particular provide a versatile tool (climber can control the angle and length of the pendulum) that can be very efficient, but can also pose risks of severe injury if impact against rock occurs after significant amount of momentum is gained. A short venture into the mathematics of pendulum dynamics will help illustrate the points.

15.1 Pendulum

The equation of motion of a simple pendulum (length l) can be derived from either the conservation of angular momentum about the hinge point, or the conservation of mechanical energy, yielding

$$I\ddot{\theta} + mgl\sin(\theta) = Q, \tag{15.1}$$

where $I = ml^2$ is the moment of inertia, θ the pendulum angle from the vertical, g the gravitational acceleration, and Q an external force, typically including friction and external torques. Assuming small amplitude (applicable in rock-climbing) and neglecting Q, this nonlinear ODE can be solved easily using the $\sin\theta \approx \theta$ approximation. Given initial conditions $\theta(0) = \theta_0$ and $\dot{\theta} = 0$, we obtain the angular dependence

$$\theta = \theta_0 \cos\omega t \tag{15.2}$$

where the natural oscillation frequency $\omega = \sqrt{g/l}$, and the period

$$T = \frac{2\pi}{\omega}, \tag{15.3}$$

which is independent of θ_0.

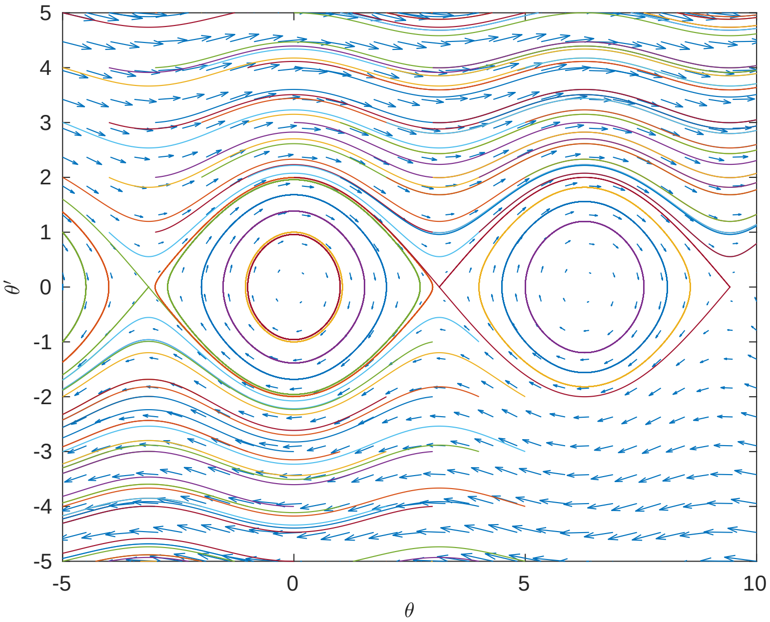

Figure 15.1: Phase portrait and vector field of a simple pendulum, with $m = g = l = 1$. Note x-axis is periodic for multiples of 2π.

The climber's (pendulum) trajectory in the phase plane (position versus velocity) is depicted in Fig. 15.1. Assuming the climber never attains energy sufficient for a full swing, he stays on the closed trajectories, typically close to $\theta \sim 0$.

Exercise 15.1 With the anchor at the apex of a mountain, the climber (weight m) abseils along a moderately-sloped mountain ridge, rather than along the vertical, due to easier terrain. He fails to place directional pieces and slips (rope length L, initial angle from the vertical θ_0).

- What is the magnitude of the torque on the anchor (hinge point) at the moment he slips?
- Suppose that during the resulting unintentional pendulum he encounters a hard wall, positioned most unfortunately, and softens the impact with his hand (area A, t). What is the impact pressure on the hand?

Solution: The magnitude of the initial torque is

$$\tau = mgl \sin\theta_0. \tag{15.4}$$

Fig. 15.1 shows that his momentum (velocity) reaches maximum at the vertical ($\theta = 0$), which is the most unfortunate position for a hard obstacle to encounter. We also observe that the momentum vanishes only at the maximum deviation ($\theta = \theta_{max}$).

From the conservation of mechanical energy (conversion of gravitational potential energy into kinetic energy),

$$mgh = \frac{1}{2}mv^2, \tag{15.5}$$

and the trigonometric relationship

$$h = L(1 - \cos\theta_0), \tag{15.6}$$

we obtain the change in velocity v for a given loss of height h,

$$v = \sqrt{2gh}. \tag{15.7}$$

During the impact the velocity rapidly decays to zero, yielding decelaration $a = v/t$. As the climber soften the impact during time t, the resulting force $F = ma$ is distributed across the area A of his hand.

Using order-of-magnitude estimates $A = 100$ cm^2, $L = 60$ m, $\theta_0 = \pi/4$, $m = 100$ kg and $t = 1$ ms as representative values, we obtain torque $\tau \sim 40$ kN·m on the anchor, and the pressure $p \sim 200$ MPa on the hand, which is on the order of the break point for human joints. If the climber can use friction prior the impact, he can reduce the impact velocity and and increase the deceleration distance.

■

15.2 Tension Traverse

Tension traverse is often used by the leader for lateral movement. The advantage is that the taut rope provides solid, unidirectional counter-pressure, that opens up new possibilities for additional footholds and handholds. The disadvantage is the need for repeated communication with the belayer ("tension/slack") and the associated delay in climbing progress.

15.3 Traverse Rescue

Tension traverses and pendulum may also complicate seconding (often requiring lower-outs) and rescue operations. Rescue of a leader who failed a traverse depends on the degree of cooperation the leader can provide:

1. A leader capable of cooperating will re-climb back to his last piece of protection before the traverse, clip in to the rope leading to the belayer (to guide his descend), and be lowered back across the traverse.
2. A leader incapable of cooperating (e.g. difficult terrain or injury) will need to be clipped in to the rescue rope (and subsequently lowered across the traverse) by another person, e.g. the belayer. The belayer must tie off the climber (fix the rope, don't rely on friction knot alone and back it up) and then reach the leader by a combination of Tyrolean traverses and/or prusiking down.

16. Rope Entrenchment

The solution to the rope becoming entrenched during abseil depends on the given rope configuration. In order of increasing difficulty and hazards, we can classify the rope configurations and the associated solutions as follows:

- Both end down Solution: Prusiking
- Only one end down, residual length sufficient Solution: Leading (belayed)
- Only one end down, residual length insufficient Solution: Leading (solo), e.g. with clove-hitch self-belay (if no specialized rescue gear is available).

R To avoid stuck ropes:

- Have the rope coils attached to your harness (e.g. with a sling) during abseil to reduce chances of rope entrenchment.
- Extend the abseil point to avoid high-friction spots.
- Split a long abseil over multiple short abseils (divide and conquer).

17. Aid Climbing

Aid climbing is an essential technique in alpine terrain and big walls, as it allows you to overcome pitches that are difficult/impossible for you to climb free. We only give a brief overview of the essentials in a minimalist situation, with emphasis on physics.

17.1 Common Methods

17.1.1 Leading

Typical leading gear includes two daisies attached to your harness. The other end of each daisy is attached to a carabiner with an aider (in minimalist situations, the aiders can be improvised from slings), creating a connected daisy/aider (DA) combination. Besides the usual nuts and cams, specialized POPs may come in handy (copperheads, skyhooks). The traditional sequence of leading moves is

1. Place POP (top) and and clip DA to it.
2. Test POP.
3. Transfer weight to POP.
4. Clip rope to last POP (bottom) and remove DA from it.
5. Ascend the aider and repeat the sequence.

An alternative is to clip to rope to the top POP (after it was tested), see Sec. 17.2 for details.

Upon climbing the pitch, the leader sets up a belay station and a hauling station.

17.1.2 Following

The follower prusiks up the rope fixed by the leader using jumars (ascenders) and DAs, while the leader hauls up the baggage on a dedicated haul rope. ’Lower-outs’ should be performed when following pendulums or tension traverses.

17.2 Common Mistakes

17.2.1 Double ropes vs Two daisy chains

Traditionally, the two daisies allow the leader to self-regulate the tension and body position during the climb and POP manipulation. An alternative is to use two ropes, and instruct the belayer to increase the tension and/or slack as needed by issuing appropriate commands (typically verbally). This alternative, however, requires constant communication between the climbers, which is often impractical or even impossible in alpine conditions. More importantly, however, having the belayer repeatedly adjust the tension in the two ropes puts an unnecessary load on the belay system, increasing the risk of POPs failure.

17.2.2 Rope clipping while leading

The traditional method of clipping the rope to the bottom POP is safer, as this POP was more rigorously tested, namely by the full weight of the climber having climbed along the attached aider. The alternative method of clipping the rope to the top POP may be more convenient, but in case the top POP fails the fall factor is higher, and so is the impact force on the other POPs. This increases the risk of serial failure of multiple POPs ('zipping-out'), and thus the risk of bodily harm.

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